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Constant strain-rate compression test of a fluid-saturated poroelastic sample with positive or negative Poisson's ratio

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Shorten Title: Compression test of a poroelastic sample

Summary. Quasi-static behavior of a poroelastic circular cylinder subjected to constant strain-rate, axial, compression was analyzed within the framework of the continuum mechanics of fluid-saturated, linear, poroelastic materials. The solutions obtained in the Laplace space were numerically inversed into the time domain. The results were illustrated in the form of curves for a wide range of drained Poisson's ratio (from positive to negative). It was found that, although the axial strain increases linearly with respect to time, all other strains, pore fluid pressure and all stresses do not; especially, in the case of the negative Poisson's ratio, the tangential strain becomes tensile immediately after loading and reverses its sign after a while; thereafter the compressive strain grows up to infinity. The pore fluid pressure and the tangential stress approach a steady state (constant) value after experiencing their transient process. Apparent stress-strain curves are nonlinear due to the pore fluid diffusion, with the more nonlinear for the smaller Poisson's ratio, especially for the negative ratio.

1. Introduction

In our previous paper [1], we treated a foam with negative Poisson's ratio [2] within the framework of the continuum mechanics of fluid-saturated poroelastic materials [3]. That is, we analyzed two-dimensional and three-dimensional, Mandel [4] and Cryer [5] problems for such a novel material as to have negative Poisson's ratio. These problems were motivated by our interest in how the foam with the negative Poisson's ratio mechanically behaves for step-like loading if it was saturated with fluid. The analysis gave some interesting conclusions as follows: (1) The peak of pore fluid pressure at a sample center is higher for the three-dimensional problems than for the two-dimensional ones and for the Cryer problem than for the Mandel Problem. (2) The Mandel-Cryer effect is much more remarkable for the smaller Poisson's ratio (especially for the negative ratio) for all of the four problems. (3) Near the sample surface, the pore fluid pressure declines from its initial, undrained, elastic response and then returns to its final, drained, elastic state. To compensate this decline during compaction, the pressure deep within the sample climbs up, reaches its peak, and then dissipates to the final state. This behavior is common for the Mandel and Cryer problems. (4) A similar behavior as in the pressure can be seen for the tangential stress in the Cryer problems and for the axial and tangential stresses in the Mandel problems. (5) The cylindrical samples for the three-dimensional Mandel problem expands instantly after loading, then gradually shrinks, and finally becomes thinner than its original thickness if the cylinder material has negative Poisson's ratio.

The tackling of these problems in [1] was motivated by the interest mentioned above. Motivation of the present study is the same as in the previous one; the interest in how a circular cylindrical sample of fluid-saturated, poroelastic material with negative Poisson's ratio behaves when the cylinder is subjected to axial compression with a constant strain-rate, instead of step-like loading in the previous paper. We will here again pick up the cylinder problem similar to the previous one but different in loading type, because this assumption of the constant strain-rate is sounder than the step-like loading when the compression tests are treated.

In a compression test problem of a "purely elastic" circular cylinder under constant strain-rate axial loading, all stress components induced are zero except the axial stress component, which is uniform on any cross section and increases linearly with respect to time; all normal strain components also grow linearly with time. In contrast, all stress and strain components except the axial strain nonlinearly depend on both the radial coordinate and time

in the present problem of a fluid-saturated poroelastic cylinder under constant strain-rate compression; this coordinate dependence is expected to be caused by radial diffusion of the pore fluid. We will focus on this apparent nonlinear behavior.

2 Constant strain-rate compression test

A circular cylinder of fluid-saturated, isotropic, poroelastic material is stress-free and freely drains on its lateral surface and subjected to axial compression of constant strain-rate on its upper and lower ends; it has a radius of a and the strain rate is denoted by $\dot{\epsilon}_0$. This problem does not require the cylinder's length because all stresses, strains and pore fluid pressure do not vary along the axial direction. We will use the cylindrical polar coordinate system (r, θ, z) , where the z -axis coincides with the cylinder axis.

It is easy to understand that non-zero components of displacement, strain, stress and pore fluid pressure are expressed as follows:

$$u_r = u(r, t), \quad u_z = \dot{\epsilon}_0 z t H(t), \quad \sigma_r = \sigma_r(r, t), \quad \sigma_\theta = \sigma_\theta(r, t), \quad \sigma_z = \sigma_z(r, t), \quad (1)$$

where the loading condition of compression of constant strain-rate has been considered, and

$$\epsilon_r = \epsilon_r(r, t), \quad \epsilon_\theta = \epsilon_\theta(r, t), \quad \epsilon_z = \epsilon_z(r, t), \quad p = p(r, t), \quad (2)$$

with $H(t)$ being the Heaviside unit step function of time t . The boundary conditions for the present situation are given by

$$r = a: \quad \sigma_r = 0, \quad p = 0. \quad (3)$$

The former is the stress-free condition while the latter the free-drain condition at the cylinder surface.

3 Basic equations

The basic equations for the mechanics of fluid-saturated, isotropic, linear, poroelastic materials in the case of non-rotational displacement field are given in the previous paper [1]. However, to make the present description complete, we will summarize these equations here again [3].

It is easy to see from Eq. (1) that the problem satisfies this irrotational condition. If total stress, pore fluid pressure, average strain, and average displacement are denoted by $\bar{\sigma}_{ii}$, p , $\bar{\epsilon}_i$ and $\bar{u}_i$, the constitutive equations, equilibrium equations for the quasi-static case of no body force and the strain-displacement relations are given by

$$\sigma_{ij} = 2G\epsilon_{ij} + \frac{\alpha}{1-2\alpha}\sigma_{kk}\epsilon_{ij} - \alpha p\delta_{ij}, \quad (4)$$

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0, \quad (5)$$

$$\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad (6)$$

where the summation convention on repeated indices is applied; G and α are the shear modulus and Poisson's ratio in the drained state; β is Biot's coefficient of effective stress given by

$$\beta = \frac{3(\alpha_u - \alpha)}{B(1 + \alpha_u)(1 - 2\alpha)}, \quad (7)$$

with B and α_u being Skempton's pore fluid pressure coefficient and Poisson's ratio in the undrained state.

Eqs. (4)-(7) hold also for the rotational displacement field. The following diffusion equation of pore fluid is valid only in the irrotational displacement case:

$$\frac{\partial p}{\partial t} = c\alpha^2 p \nabla^2 \frac{d}{dt} g(t), \quad (8)$$

with the storage coefficient S and the diffusivity coefficient c given by

$$S = \frac{\alpha^2(1-2\alpha)^2(1-\alpha_u)}{2G(\alpha_u - \alpha)(1-\alpha)}, \quad c = \frac{\alpha}{S}, \quad (9)$$

where β is the permeability coefficient or mobility coefficient; $g(t)$ is an auxiliary function of only t . This function is related to volumetric strain and pore fluid pressure as follows:

$$\epsilon_{kk} = \frac{\alpha}{G} p + g(t), \quad (10)$$

where

$$\alpha = \frac{\alpha(1-2\alpha)}{2(1-\alpha)}. \quad (11)$$

4 Solutions in terms of $g(t)$ and $p(r, t)$

It follows from Eq. (10) that

$$\epsilon_r(r, t) + \epsilon_\theta(r, t) + \epsilon_z(t) = \frac{\alpha}{G} p(r, t) + g(t) \quad (12)$$

in the cylindrical polar coordinate system. Using this equation in the expression for $\epsilon_r(r, t)$ obtained from Eq. (4) yields

$$\epsilon_r(r, t) = 2G\epsilon_r(r, t) - 2\alpha p(r, t) + \frac{2G\alpha}{1-2\alpha} g(t), \quad (13)$$

where Eq. (11) has been used. Reuse of Eq. (4) together with Eq. (6) gives

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (ru(r,t)) \right) = 2G \frac{\partial u(r,t)}{\partial r} - \frac{u(r,t)}{r}. \quad (14)$$

Substituting Eqs. (13) and (14) into the equilibrium equation (5) expressed in the cylindrical polar coordinate system and arranging the resulting equation yield

$$\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (ru(r,t)) \right) = \frac{1}{G} \frac{\partial p(r,t)}{\partial r}. \quad (15)$$

Indefinite integration of the above equation, which yields an integral constant $C_1(t)$, and further integration of the resulting equation from 0 to r with respect to r gives

$$u(r,t) = \frac{1}{G} \frac{1}{r} \int_0^r p(r,t) r dr + C_1(t)r. \quad (16)$$

From use of this displacement in Eq. (6) rewritten in the cylindrical coordinate system, it follows that

$$\begin{aligned} \epsilon_r(r,t) &= \frac{1}{G} \left[p(r,t) - \frac{1}{r^2} \int_0^r p(r,t) r dr \right] + C_1(t), \\ \epsilon_\theta(r,t) &= \frac{1}{G} \frac{1}{r^2} \int_0^r p(r,t) r dt + C_1(t). \end{aligned} \quad (17)$$

Substituting the above equations into Eq. (4) in the cylindrical polar coordinate system yields

$$\begin{aligned} \sigma_r &= 2G \left[C_1(t) + \frac{1}{1+2\nu} g(t) - (2\nu) \frac{1}{r^2} \int_0^r p(r,t) r dr \right], \\ \sigma_\theta &= 2G \left[C_1(t) + \frac{1}{1+2\nu} g(t) + (2\nu) \frac{1}{r^2} \int_0^r p(r,t) r dr - p(r,t) \right]. \end{aligned} \quad (18)$$

Applying the first boundary condition in Eq. (3) to the first equation in Eqs. (18) determines the integral constant:

$$C_1(t) = -\frac{1}{1+2\nu} g(t) + \frac{1}{G} \frac{1}{a^2} \int_0^a p(r,t) r dr. \quad (19)$$

Thus, if this integral constant obtained is substituted back into Eqs. (17) and (18), all the radial and tangential components of strain and stress are expressed in terms of $g(t)$ and $p(r,t)$.

From use of these obtained strain components in Eq. (12) and furthermore in Eq. (4), the axial strain and axial stress are also obtained in terms of $g(t)$ and $p(r,t)$ as follows:

$$\epsilon_z(t) = \frac{1}{1+2\nu} g(t) - \frac{2\nu}{G} \int_0^a p(r,t) r dr, \quad (20)$$

$$\sigma_z(r,t) = \frac{2G(1+\nu)}{1+2\nu} g(t) - (2\nu) \left[\frac{2}{a^2} \int_0^a p(r,t) r dr + p(r,t) \right]. \quad (21)$$

5 Final solutions in Laplace space

In this stage, the auxiliary function $g(t)$ and the pore fluid pressure $p(r,t)$ are still unknown.

To obtain the latter, applying the Laplace transform to Eq. (8) written in the cylindrical polar coordinates, solving the resulting ordinary differential equation and considering the second boundary condition in Eq. (3) and the pressure regularity at the cylinder center produce

$$\bar{p}(r, s) = \frac{\bar{g}(s)}{s} \frac{I_0(\sqrt{s/cr})}{I_0(\sqrt{s/ca})}, \quad (22)$$

where the over bar on the dependent variables means their Laplace transform with respect to time with its subsidiary variable s . Substituting this solution into Eq. (20) after transformation and then applying the last equation in Eqs. (1) give $\bar{g}(s)$ as follows:

$$\bar{g}(s) = \frac{(1 - 2\nu)(1 - \nu_u)}{s^2 (1 - \nu)(1 - \nu_u) \frac{2I_1(\sqrt{s/cr})}{(\sqrt{s/ca})I_0(\sqrt{s/ca})}}. \quad (23)$$

Now that we have obtained the pore fluid pressure and the auxiliary function, we can write all necessary solutions. In order to represent all of them, we will introduce the following non-dimensional parameters and function:

$$\bar{t} = t/(a^2/c), \quad \bar{s} = s(a^2/c), \quad \bar{r} = r/a, \quad D(\bar{r}) = (1 - \nu)(1 - \nu_u) \frac{2I_1(\sqrt{\bar{r}})}{\sqrt{\bar{r}}I_0(\sqrt{\bar{r}})}. \quad (24)$$

By use of these non-dimensional variables, the desired solutions in their non-dimensional forms are fully determined. The pore fluid pressure and stresses are given by

$$\frac{\bar{p}(\bar{r}, \bar{t})}{2G_0(a^2/c)^2} = \frac{B}{3} \frac{(1 - \nu)(1 - \nu_u)}{\bar{t}^2 D(\bar{r})} \frac{I_0(\sqrt{\bar{r}})}{I_0(\sqrt{\bar{r}})}, \quad (25)$$

$$\frac{\bar{\varepsilon}_z(\bar{r}, \bar{t})}{2G_0(a^2/c)^2} = \frac{1}{\bar{t}^2 D(\bar{r})} \frac{(1 - \nu)(1 - \nu_u)}{(1 - \nu)(1 - \nu_u)} \frac{I_0(\sqrt{\bar{r}})}{I_0(\sqrt{\bar{r}})} + \frac{2I_1(\sqrt{\bar{r}})}{(\sqrt{\bar{r}})I_0(\sqrt{\bar{r}})}, \quad (26)$$

$$\frac{\bar{\sigma}_r(\bar{r}, \bar{t})}{2G_0(a^2/c)^2} = \frac{\nu_u}{\bar{t}^2 D(\bar{r})} \frac{I_0(\sqrt{\bar{r}})}{I_0(\sqrt{\bar{r}})} - \frac{I_1(\sqrt{\bar{r}})}{(\sqrt{\bar{r}})I_0(\sqrt{\bar{r}})} - \frac{I_1(\sqrt{\bar{r}})}{(\sqrt{\bar{r}})I_0(\sqrt{\bar{r}})}, \quad (27)$$

$$\frac{\bar{\sigma}_r(\bar{r}, \bar{t})}{2G_0(a^2/c)^2} = \frac{\nu_u}{\bar{t}^2 D(\bar{r})} \frac{I_1(\sqrt{\bar{r}})}{(\sqrt{\bar{r}})I_0(\sqrt{\bar{r}})} - \frac{I_1(\sqrt{\bar{r}})}{(\sqrt{\bar{r}})I_0(\sqrt{\bar{r}})}. \quad (28)$$

To illustrate the stress-strain curve, we need the axial stress $\bar{\sigma}_{zm}(\bar{t})$ averaged over the cross-section, which comes from Eq. (26) as

$$\frac{\bar{\sigma}_{zm}(\bar{t})}{2G_0(a^2/c)^2} = \frac{1}{\bar{t}^2 D(\bar{r})} \frac{(1 - \nu)(1 - \nu_u)}{(1 - \nu)(1 - \nu_u)} \frac{4I_1(\sqrt{\bar{r}})}{(\sqrt{\bar{r}})I_0(\sqrt{\bar{r}})}. \quad (29)$$

Since the average axial stress complicatedly depends on time as seen from the above equation, it is easy to predict the stress-strain curve is apparently non-linear. The Poisson's ratio, defined by the ratio of a lateral strain to an axial strain, depends on time; let us call this ratio an "apparent Poisson's ratio." The lateral strain, i.e., the tangential strain in this case, is given by

$$\frac{\bar{u}_H(\bar{t}, \bar{D})}{\bar{D}_0(a^2/c)^2} = \frac{1}{\bar{D}^2 D(\bar{D})} \left[1 - \bar{D} \bar{D}_u(\bar{D}) \right] \left[\frac{I_1(\sqrt{\bar{D}} \bar{D})}{(\sqrt{\bar{D}} \bar{D}) I_0(\sqrt{\bar{D}})} + \frac{I_1(\sqrt{\bar{D}})}{(\sqrt{\bar{D}}) I_0(\sqrt{\bar{D}})} \right]. \quad (30)$$

It should be mentioned here that in the above non-dimensional solutions (25)-(30) their inverse, for example $\bar{u}_H(\bar{t}, \bar{D})$, is defined as

$$\frac{\bar{u}_H(\bar{t}, \bar{D})}{\bar{D}_0(a^2/c)^2} = \frac{1}{2\pi i} \int_{Br} \frac{\bar{u}_H(\bar{t}, \bar{D})}{\bar{D}_0(a^2/c)^2} e^{\bar{D} \bar{t}} d\bar{D}, \quad (31)$$

where Br means the first Bromwich path; note that the over bar means the Laplace transform with respect to time t , not to the non-dimensionalized time $\bar{t}$.

6 Numerical Laplace inversion

We now have to inverse the Laplace transform solutions into the time domain. This will be done by Stehfest's numerical Laplace inversion method [6], [7], which is based on a sample of the time function using a delta-convergent series. The resultant series approximation takes the following form:

$$f(\bar{t}) \approx \frac{\ln 2}{\bar{t}} \sum_{n=1}^N c_n \bar{f}\left(\frac{n \ln 2}{\bar{t}}\right), \quad (32)$$

where the coefficient c_n is given by

$$c_n = (-1)^{n+N/2} \sum_{k=(n+1)/2}^{\min(n, N/2)} \frac{k^{N/2} (2k)!}{(N/2 - k)! k! (n - k)! (2k - n)!}, \quad (33)$$

where the number of terms N in the series must be even. Also it must be less than or equal to 20 according to [7]. We will take $N = 10$ (see [8]). Since c_n 's do not depend on $\bar{f}(t)$, we can calculate them before calculating all stress and strain components.

7 Results and discussion

Bulk modulus of almost all foams K is much smaller than those of their own skeletal polymer or metal K_s and of saturating pore fluid K_f if it is water or oil; for example, $K \approx 10$ kPa for polyester foam, $K_s \approx 1.6$ MPa for polyester itself and $K_f \approx 2.2$ MPa for water. Therefore, the incompressible ingredients model can be applied to the polyester foams saturated with water. This may be true for most liquid-saturated foams, so that we assume $\bar{\nu}_u = 0.5$ and $B = 1$ in the following numerical computation and all figures. We also assume $\bar{\nu} = 0.8$ in many figures, because this is the smallest value among those of the foams that have been fabricated until

now. In seeing the dependence on Poisson's ratio, we take $\nu = 0.4, 0.0, -0.4, -0.8$ in the figures.

In all of the following figures, all components of stress and strain appearing on the ordinate have a minus sign before them because most of them is negative almost at any time due to the axial compression. It should be also careful that all the stress and pore fluid pressure are non-dimensionalized by $2G\epsilon_0(a^2/c)$; whether we should use $2G$ or $E = 2G(1 + \nu)$ may be controversial, when we focus our attention on the dependence of the stress or the pressure on Poisson's ratio. We will, however, show only the curves non-dimensionalized by use of $2G$, because we will mainly take care of their non-linear dependence on time.

Figure 1 shows the time history of the axial stress σ_z at the sample surface $r = 1$ for various values of Poisson's ratio, while Fig. 2 does the same for $\nu = -0.8$ at various radial positions. Although the axial strain increases linearly with time due to the constant strain-rate, the axial stress does not except when $\nu = 0.5$; in this case the linear dependence on time can be seen from Eq. (26) with Eq. (24) if substituting $\nu = 0.5$ and $\epsilon_u = 0.5$ into them. It is seen from Fig. 1 that nonlinearity in the time dependence of the axial stress is stronger for the smaller Poisson's ratio. From Fig. 2 and also from the limits of $t \rightarrow 0+$ and $t \rightarrow \infty$ in Eq. (26), it is seen that the initial slope is steeper at the more inner position and that the final slope becomes the same constant value for every position on the cylinder cross-section.

Figure 3 depicts the time history of the tangential strain ϵ_θ at the sample surface $r = 1$ for the four different values of Poisson's ratio. It should be pointed out that, although the strain starts with the same slope for all the distinct ratios, the strain varies differently depending on Poisson's ratio; if $\nu > 0$, the cylinder surface extends in its circumferential direction and grows further together with time. In contrast, if $\nu < 0$, the surface suffers extension at an early stage, stops to extend after a while, thereafter changes sign in the strain slope and finally experiences tangential contraction, which tends to infinity for $t \rightarrow \infty$. When just $\nu = 0$, the surface extension remains constant after the initial extension, which is understood from the limit of $t \rightarrow \infty$ in Eq. (30). The same strain history for $\nu = -0.8$ is shown in Fig. 4 for various radial positions. The maximum extension occurs at the center of cylinder and the slope after a long time seems the same everywhere within the cylinder; this is true, as seen from Eq. (30).

The time history of the pore fluid pressure p is shown in Figs. 5 and 6; the former is the one at the cylinder center when Poisson's ratio takes the four typical values, while the latter for various radial positions when $\nu = -0.8$. It is seen from these figures that the pressure

Fig. 1. Time history of axial stress at cylinder surface for four different Poisson ' s ratios

Fig. 2. Time history of axial stress at various radial positions for $\nu = -0.8$

Fig. 3. Time history of tangential strain at cylinder surface for four different Poisson ' s ratios

Fig. 4. Time history of tangential strain at various radial positions for $\nu = -0.8$

Fig. 5 Time history of pore fluid pressure at cylinder center for four different Poisson's ratios

Fig. 6 Time history of pore fluid pressure at various radial positions for $\nu = -0.8$

starts with zero, transiently increases and finally converges to a certain time-independent value, which is given by the limiting process of Eq. (25) as follows:

$$\frac{p(\bar{r}, \infty)}{2G\bar{r}_0(a^2/c)} = \frac{B(1-\bar{\nu})(1+\bar{\nu}_u)}{12(1-\bar{\nu}_u)}(1-\bar{r}^2). \quad (34)$$

This converging value depends on B , $\bar{\nu}$, $\bar{\nu}_u$ and $\bar{r}$ as above, although we have here taken $B=1.0$, $\bar{\nu}_u=0.5$ (the incompressible ingredient model). The higher it is, the smaller the Poisson ratio, and $p(0, \infty) = 0.125\bar{\nu}\{2G\bar{r}_0(a^2/c)\}$, even when $\bar{\nu}=0.5$. This converging behavior is completely different from that of the axial stress and tangential strain shown in Figs. 1 to 4, where these components diverge infinitely as time elapses.

There is no peak of the pore fluid pressure at any position for any Poisson's ratio in either figure; that is, the Mandel-Cryer effect cannot be expected for the case the cylinder is loaded by compression of the axial constant strain-rate, instead of the step-like loading in the Mandel and Cryer problems.

The tangential strain history has been shown in Figs. 3 and 5, while the history of the tangential stress is given in Figs. 7 and 8. From comparison between these two pairs of figures, even the same components of strain and stress are very different in their behavior. Fig. 7 depicts the history of the tangential stress at various radial positions. Their values converge to a certain time-independent one, which depends on the position $\bar{r}$; it is compressive at the inner position but tensile in the outer position. The dependence of the history on Poisson's ratio can be seen in Fig. 8; as can be also seen in Eq. (27), no tangential stress appears when $\bar{\nu}=0.5$ and the higher tensile stress occurs for the more negative Poisson ratio. Such a dependence is given by

$$\frac{\bar{\sigma}_\theta(\bar{r}, \infty)}{2G\bar{r}_0(a^2/c)} = \bar{\nu} \frac{\bar{\nu}_u \bar{\nu}}{16(1-\bar{\nu}_u)}(1-3\bar{r}^2), \quad (35)$$

which has been obtained from Eq. (27) by the limiting process. From this equation, it can be seen that $\bar{\sigma}_\theta(\bar{r}, \infty) = 0$ at $\bar{\nu}=1/\sqrt{3}$ for every $\bar{\nu}_u$ and $\bar{r}$.

We have already obtained the axial stress averaged over the cylinder cross-section $\bar{\sigma}_m$, given by Eq. (29). The averaged axial stress-axial strain and the averaged axial stress-tangential (lateral) strain relations are depicted in Fig. 9; the former is drawn by solid curves while the latter by broken lines. The tangential strain $\bar{\epsilon}_\theta$ at the cylinder surface $\bar{r}=1$ is drawn in the figure, because it is measured there by strain gage in many experiments. The average axial stress can be obtained from the axial load measured and recorded in experiments.

From Fig. 9, it is found that the solid and broken curves are rather linear when Poisson's ratio is positive ($\bar{\nu}=0.4$), while these are more nonlinear for the more negative

Fig. 7 Time history of tangential stress at various radial positions for $\nu = -0.8$

Fig. 8 Time history of tangential stress at cylinder surface for four different Poisson ' s ratios

Fig. 9 Stress-strain curves for axial and tangential strains for four different Poisson ' s ratios

Fig. 10 Time history of apparent Poisson ' s ratio for four different Poisson ' s ratios

($\nu = 0.4, 0.8$); especially the lateral strains drawn by broken curves are negative (opposite sign for stress and strain) for smaller average axial stress and turn to positive (the same sign) as the axial stress grows. In general, the more nonlinear the curves are, the smaller or the more negative the Poisson ratio is.

From the curves in Fig. 9, we can draw other curves depicting the time history of "apparent Poisson's ratio," defined by

$$\nu(\bar{\epsilon}) = - \frac{\bar{\epsilon}_r(1, \bar{\epsilon})}{\bar{\epsilon}_z(\bar{\epsilon})}, \quad (36)$$

which is shown in Fig. 10. Before discussing the curves in this figure, we should remember that the two Poisson's ratios, ν_u and ν_l , are defined as ratios of a lateral strain to axial strain for a uni-axial tension in the undrained and drained states, respectively. Therefore, we can predict that, if a fluid-filled poroelastic cylinder is subjected to step-like axial loading (i.e., the three-dimensional Mandel problem), its apparent Poisson's ratio is ν_u at the instant immediately after loading and ν_l after sufficiently long time. In the present problem, the loading is compression of the constant strain-rate given by $d\bar{\epsilon}_z(\bar{\epsilon})/d\bar{\epsilon} = \bar{\epsilon}_b H(\bar{\epsilon})$. It is not easy to intuitively foresee the initial and final apparent Poisson's ratios. From the numerical results shown in Fig. 10, it is seen that the apparent Poisson's ratios are

$$\nu(\bar{\epsilon} = 0+) = \nu_u, \quad \nu(\bar{\epsilon} \rightarrow \infty) = \nu_l, \quad (37)$$

even when the loading is not step-like. This can also be predicted by an analytical manner; the limiting process gives

$$\frac{\bar{\epsilon}_r(1, \bar{\epsilon})}{\bar{\epsilon}_b(a^2/c)} = \nu_u \bar{\epsilon} \text{ for small } \bar{\epsilon}, \text{ and } \frac{\bar{\epsilon}_r(1, \bar{\epsilon})}{\bar{\epsilon}_b(a^2/c)} = \nu_l \text{ for large } \bar{\epsilon}. \quad (38)$$

From these limiting values, Eq. (37) can be recovered, if taking into account $\bar{\epsilon}_z(\bar{\epsilon}) = \bar{\epsilon}_b(a^2/c) H(\bar{\epsilon})$ for all time.

8 Conclusions

We have solved a transient response problem of a fluid-saturated poroelastic circular cylinder (core sample) subjected to axial compression with a constant strain rate; the cylinder has a wide range of Poisson's ratio, from positive to negative. From the solutions and numerical calculations, we have obtained the following conclusions:

- (1) In spite of the uni-axial compression, the time history of all the normal stress component and pore fluid pressure depends both on the Poisson ratio and radial position.
- (2) The cylinder extends in the tangential direction immediately after loading but changes its

sign after a while and then shrinks, if Poisson's ratio is negative.

- (3) The curve of the axial stress against axial strain and tangential strain is more nonlinear for the smaller Poisson's ratio; especially for the negative ratio.
- (4) In spite of the constant strain rate loading, the apparent Poisson's ratio starts from the undrained Poisson's ratio and converges to the drained one with time elapsing.

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