

Mathematics Activities and Ideas : Grades Six to Twelve

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Introduction

As a teacher and teacher educator, the author has developed and implemented several hands-on mathematics activities and demonstration models, both in Japan and in Pakistan. This article presents a few of those activities that have provided the most successful results in learning environments. The activities are relevant to mathematical concepts and principles closely related to elementary, secondary and higher-secondary (grades six to twelve) Mathematics Curricula of schools in general. The activities described here provide guidelines for hands-on experiences for students, and are designed to make mathematics lessons as meaningful and enjoyable as possible.

In an average classroom, the teacher is usually confronted by a variety of different ways of thinking among the students. These traits have often been divided into the four more or less discrete categories vis-a-vis personality, as designated by Carl Gustav Jung: logical, sensory, emotional, and intuitive thought. It would be highly instructive, then, to provide some explanation of these personality types as they relate to the types of students we may encounter in the classroom. In this way, we can make the most of the lessons by directing them toward the students' specific learning styles.

○ The logical student can easily understand mathematical ideas by reading textbooks, because

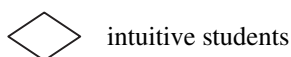
most are written in a logical way, pertaining to the concepts they describe.

- Sensory-type students sometimes feel difficulty in reading mathematics, because they do not possess adequate qualities for engaging in abstract thinking, yet they may be able to handle concrete thought suitably. In primary school, we sometimes observe the phenomenon that sensory pupils unconsciously apply some imagery to natural numbers in a simple calculation (such as shopping with mother), and find that they cannot add those numbers when presented in numerical terms.
- It would seem that the emotional students would study harder if they are fond of their teacher and feel comfortable in their learning environment. For these types of people, such factors can be conducive to learning. Conversely, if they do not like their teacher or textbook, they might not study the subject at all.
- Among intuitive students, we sometimes find a clear thinker. That student can immediately grasp the result and understand how it was derived, even while the teacher is still introducing the problem. But such a student often struggles in explaining how he arrived at the solution, in spite of having seen it clearly for himself.

From the above descriptions of thinking styles per personality, we can conclude that the teacher needs to take into account and effectively cater to the four

personality types, in order that the teaching objectives can be achieved ; this is particularly important to note, as many teachers tend to teach according to their own personality type, or by meticulously following the textbook.

The following legend pertains to the type of classroom teaching aid used with each type of personality, indicated in the lessons presented in this paper :



intuitive students



sensory students



emotional students



logical students

Mathematical apparatus, or demonstration models, are useful for those students other than the logical type. For this reason, the author has made a variety of different teaching aids or kits over the years, directed specifically toward the intuitive, sensory and emotional types of thinkers.

Ordinarily, everyday objects are the major resource for the lesson activities. It is a common experience that even in simple mathematics, commercial teaching aids are costly. Many teachers are not often in a position to purchase teaching aids in order to provide the opportunity for their students to explore mathematics in a more meaningful and enjoyable way. Ordinary objects, such as straws, wire, and plastic drinking bottles are economical, freely available and easy to use. The teaching ideas presented here are primarily for the teacher ; but it is hoped that even students, parents and the public at large will find them fun and highly interesting.

1 . Keep a Balance

Relation : Punjab Textbooks (2003)

Grade	Chapter
Primary	Arithmetic
6	
7	7.Linear Equation
8	7.Linear Equation
9	
10	
11	
12	

Purpose : For the explanation of a linear equation. For example : $4x+3=11$

Application : For the explanation of any other topic including an 'equals' sign ; for example, an arithmetic calculation :

$$37 = 10 \times 3 + 7,$$

$$\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$$

You will need :

- 1 meter of steel wire
- Three straws or a half-meter length of string
- A sheet of A-1 thick white paper and a few sheets of thick colored paper (thickness should all be the same)
- Some paper clips
- Three plastic suction cups
- Tools : two spanners or pliers, T-scale, scissors, pencil, marker

Caution : Be careful when handling the sharp tools and wire ends.

What to do : Cut the wire into three equal lengths. Bend each wire at the centre and make three balances. Cut straws and hang them from the wires, for the arms of the balances. A hole can be easily made at the end of straws.

Some clips should be hung from the straws. On the A-1 white paper, draw about 20 1-inch squares, 5 10-inch squares, and a few rectangles of 2-, 3-, and 5-square-inch areas. The colored paper is used to provide the necessary variables. Cut and hang them as stated in the equation. Tie three balances with string and set them in a line.

Challenge : With a triangle scale and clips, try to make a balance. If you can get some plastic suction cups, it is easy to hang the balance anywhere, such as from a white board or a window.

Your Turn : Could you use the balance to explain an arithmetic calculation? Try to present problems in everyday situations.

Core : A balance expresses an 'equals' sign. '=' often poses some difficulties in mathematical definitions in certain situations. Also in many cases, the average student may find it hard to comprehend the concept of an 'equals' sign and its implications. Thus, teachers sometimes need to emphasize it. This activity can be used to express the '=' sign clearly.

Personalities :



intuitive students



sensory students



logical students

2. A String of 12

Relation : Punjab Textbooks (2003)

Grade	Chapter
Primary	Basics of Figures
6	7. Geometry
7	8. Geometry
8	9. Geometry

9	7. Fundamental Geometry
10	
11	
12	

Purpose : For drawing various figures.

Application : For drawing large figures on the ground using 12 10-meter ropes.

You will need :

- 1-meter string
- 12 drawing pins
- Scotch tape

What to do : Make a loop with the 1-meter string, without a knot, by attaching the two ends together with tape. Divide it into 12 equal parts. Then, with scotch tape, attach each of the 12 pins at each point of the string.

How to use : Let's make an equilateral triangle. One side is 4, meaning 4 segments. (Be sure to count by the number of segments, rather than by the number of pins, so that students will understand this concept in terms of the sides of the polygon.) Next, form a right triangle using Pythagorean numbers 3, 4, 5. We get a right angle. Then, using the right angle, let's make a square. Each side is 3 segments. Try a rectangle (4, 2, 4, 2), a parallelogram, a rhombus, a trapezoid (5, 2, 3, 2), a quadrilateral/quadrangle (5, 4, 2, 1), and so on.

Let's make a hexagon. From the equilateral triangle, we mark 2 pins at the 2 sides. And then make an equilateral triangle on the opposite side of the base. Then, join two edges of the base and 4 marked points.

Now let's draw a circle and an oval. At the oval, fastening two pins on a big piece of paper in less than 5 distances, stretch the longer string with a pencil. Keeping the string tight, draw a curve with the pencil.

Challenge : Try to draw the 200m running track at your school ground, in preparation for an athletic

meeting, with 12 10-m ropes.

Your Turn : With this kit, try to draw a star shape.

Core : 12 is quite an important number. \triangle : (5,5,2), (4,4,4), (3,4,5) ; \diamond (4,2,4,2), (3,3,3,3), etc. There are many mysteries concerning the base-12 number system, regarding time and angle, in addition to the number of the days and months of a year. For example,

$$\begin{aligned} 365 &= 102 + 112 + 122 \\ &= 132 + 142 \end{aligned}$$

Because string is being used, rather than rigid materials such as plastic sticks, an oval can be drawn, because an oval has such characteristics that the sum of the distances from any point on the oval to the two focuses is constant.

Personalities :



sensory students



logical students



emotional students : If you can draw an interesting image, for example a heart, with this tool before your explanation.

3. Pie Graphs

Relation : Punjab Textbooks (2003)

Grade	Chapter
6	8. Information Handling
7	3. Rational Numbers 9. Information Handling
8	
9	
10	4. Information Handling
11	
12	

Purpose : To show various ratios repeatedly. For the computation of fractions.

Application : Relationship to other subjects of study.

To demonstrate angles.

You will need :

- A clear "PET" plastic drink bottle (of thin polyethylene teraphthlate)
- A metal pin
- A few sheets of colored paper
- Scissors, scotch tape

What to do : Use the PET bottle as a stand. Make a pinhole in the side of the upper part of the bottle with a pin. Then, cut colored paper into circles and make a slit along the radius of each piece. Insert them through the pin into the stand, so they are in layers. We can adjust the area of each color to show various areas depending on the problem or situation we present to the students.

Challenge : Put two straws in front of the colored paper and insert a paper pin into the centre, so that they look like the hands of a clock. Along the edge of each colored paper starting from the slit, make marks in 1-degree increments for a total of 90 degrees. You can use this kit to visually show the computation of fractions.

Your Turn : Try to use this apparatus to explain some phenomena in daily life, such as the contrast between daylight and darkness each day at various times during the year.

Core : The combination of the colors is interesting and important for our different types of learners, especially the "emotional" students.

Personalities :



intuitive students



sensory students



emotional students

4. The Area Increases or Decreases?

Relation : Punjab Textbooks (2003)

Grade	Chapter
6	7. Geometry
7	3. Rational Numbers
8	
9	
10	1. Algebraic Sentences
11	
12	

Purpose : For the exercise of area calculations.

For the explanation of the coefficient of the incline in the linear function.

Application : For the determination of a fraction.

You will need :

- Thick paper
- Pen and a scale or drawing software
- A cutter

What to do : On a thick piece of paper, draw lines as shown in Fig. 4-1, and cut the paper along the lines. Then, in front of the students rearrange the pieces in a different pattern (Fig. 4-2). Comparing the two figures, the area seems to have changed from 64 cm^2 to 65 cm^2 .

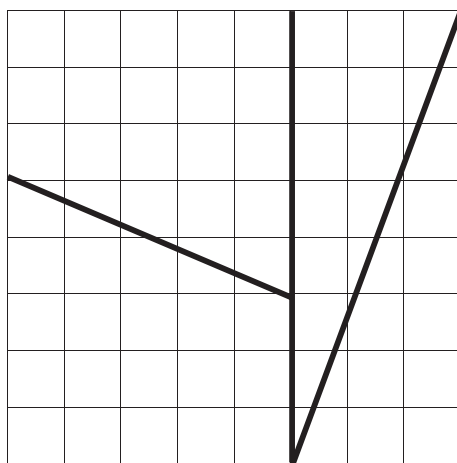


Fig.4-1

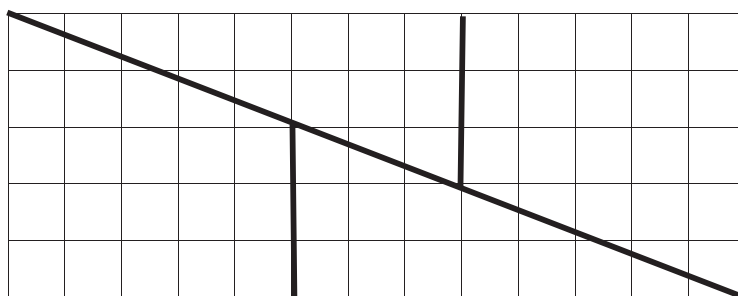


Fig.4-2

In the process of discovering the reason for the increase in area, the students will study the above purpose. On the other hand, Figure 4-3 presents

another question. We also draw and cut it along the lines. In this question, the area will decrease from 64 cm² to 63 cm².

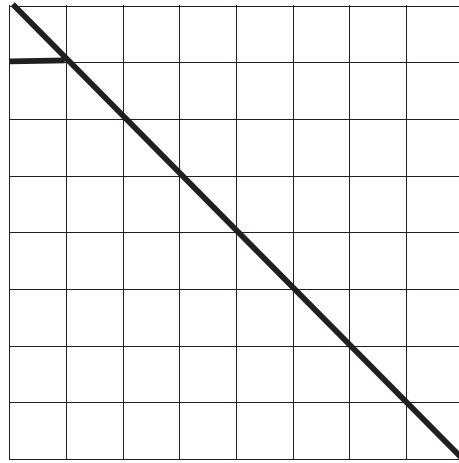


Fig. 4-3

First remove the smaller triangle of the upper left in Fig. 4-3 to the lower right. Then, slide the larger triangle of the upper right to the same bottom line

along the bold line into Fig. 4-4. We get a rectangle whose height is 7 and the width appears to be 9.

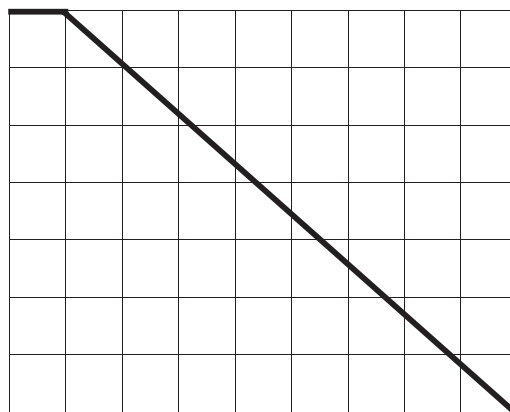


Fig. 4-4

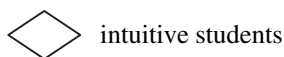
Challenge : Try to make the same kind of figure.

Your Turn : Display the activity on the OHP, and ask class members to suggest their own arrangement of the pieces.

Core : First, students should know that the area does not change due to the movement of any figure on the two-dimensional plane. For the first figure, it is

important if they can find the difference between two inclines, and in the second example, to determine the fraction of the bottom of the smaller triangle.

Personalities :



intuitive students



sensory students



logical students



emotional students

5. Cellophane Venn Diagram

Relation : Punjab Textbooks (2003)

Grade	Chapter
6	
7	
8	
9	1. Set
10	
11	
12	

Purpose : For the explanation of Sets.

Application : For the explanation of complementary colors.

Background : In color science, two colors are called complementary if, when mixed, they produce a shade of grey or white. Complementary colors are those that lie opposite each other on the color wheel (for example, red and green are complementary).

You will need :

- 4 different colored cellophane sheets (two complementary colors and two similar colors)
- Thick, different colored paper (same colors as cellophane above)
- Thick white paper
- Glue and scotch tape
- Scissors and a cutter

Caution : The edge of the thick paper is very sharp,

especially after cutting it with the cutter. So when you fold the paper, be careful not to cut your finger.

What to do : Select four colors, two which are complementary colors (for example, red and green, or yellow and blue), and two which are similar (for example, blue and green, or red and orange). Cut four sheets of cellophane into rectangles, and the remainder like a general Venn Diagram. Edge the small rectangles and the remainders with the same colored thick paper. Glue small pieces of tape on each side of the rectangles. Affix two magnets or strings on the left and right upper vertex of the rectangles.

Next, make number cards as elements of a set with the same colored paper.

Challenge : Add one more pair of cellophane sheets.

Then, make set C and C^c

Your Turn : First, put green colored numbers on the white paper of the Universal set. On these put the red large rectangle cellophane as the complement of the set A. Then, add the small blue rectangle on those as the set B. In this case, two elements 5,7 disappear, because they are complimentary colors. So, you can ask your students what the elements of $A^c \cap B$ are.

Could you explain $(A \cap B) \cup (A^c \cup B^c) \cap U$?

Core : By making the image disappear with the complementary colors, the students can clearly see the relationship, by cause and effect.

Personalities :



intuitive students



sensory students



emotional students

6. Magical Table

Relation : Punjab Textbooks (2003)

Grade	Chapter
primary	Addition
6	
7	
8	
9	
10	1. Algebraic Sentences
11	6. Sequences & Series
12	

Purpose : For the study of Fibonacci progressions.

For the study of algebraic expressions.

Application : For the exercise of addition.

You will need :

- A4 paper
- A chart, as in Fig. 6-1

	trial 1	2	3	4	5	6	7	8	9	10
line 1										
2										
3										
4										
5										
6										
7										
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										

Fig. 6-1

What to do :

First, write a number in the box on line 1 (see Fig. 6-1), and then a higher (ascending) number in the box on line 2. The teacher then tells the students what the

number will be in line 17! The students then perform the calculations, and understand that the answer is correct (Fig. 6-2).

	trial	2	3	4	5	6	7	8	9	10	
line	1										
1											a
2											b
3											a+b
4											a+2b
5											2a+3b
6											3a+5b
7											5a+8b
8											8a+13b
9											13a+21b
10											21a+34b
11											34a+55b
12											55a+89b
13											89a+144b
14											144a+233b
15											233a+377b
16											377a+610b
17											610a+987b

Fig. 6-2

Put other numbers into lines 1 and 2, and consider how to calculate the number of line 17.

Solution : The number of line 17 will be :

$$610a + 987b = 10(61a + 98b) + 7b$$

if the number of line 1 is a, and the number on line 2 is b.

Personalities :



intuitive students



sensory students



logical students



emotional students

References

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- Fujimoto, C. (2003) Mathematics Kits. Pakistan : National Institute of Science and Technical Education