Ann. Rep. Fac. Educ., Iwate Univ., Vol. 51 No. 2 (Feb. 1992) 65~73

Stress Analysis of T-Flange Bolted Joint with a Simplified Spring and Beam Model

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(Received October 15, 1991)

1. Introduction

Bolted joints are used in every type of mechanical structures such as the most simple to the most complex and expensive machinery. It is important for designers, therefore, to know the behaviour of the bolted joint. An example of the behaviour is as follows: An external force is applied on the bolted joint at the point away from the axis of the bolt, the bending stress as well as axial stress is produced in the bolt by a lever action. For theoretical analysis of the bolted joint behaviour, it is a well known technique that the bolted joint is modeled using beam and spring. In the analysis using this model, the bending moment acting on the bolt has been neglected so far for the sake of a conventional approximation[1),2)]. For the strength design of the bolted joint, however, it may be important to take into account the bending stress in the bolt, for designing machinery.

The purpose of this study is to develop a simple design method of the bolted joint by behaviour into account the bending moment, which may be applicable to general cases. In this report, the T-flange bolted joint is chosen for investigating the multi-bolted joint problem. The T-flange bolted joint is represented by only one bolt and is modeled using beam and spring. The stress behaviour in the bolt is analyzed including the effect of the bending moment acting on the bolt.

The under conditions of experiments, which is an usual method for simplifying the model, are performed using two sizes of the bolt and two stepwise preload. The experimental results are discussed by comparing with the calculated data obtained by present model corresponding to the present experiments.

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2. Analysis

Figure 1 shows the T-flange bolted joint with an applied load F_B. Figure 2 shows a beam and spring model of this joint. In the present model, the T-flange bolted joint is assumed to be symmetrical to both the loading axis and the contact suface as shown in Fig. 1. In Fig. 2, both the bolt and the compressed part of the flange are modeled by springs and also the flange is modeled by the beam. When the load is applied to the T-flange bolted joint, the reaction force is distributed widely in the contact surface by a lever action. However, the distributed reaction force is assumed as a concentrated force acting at a point apart from the center of the bolt hole by s mm. When the load increases, the concentrated reaction force shifts to the edge of the flange. At the point where the

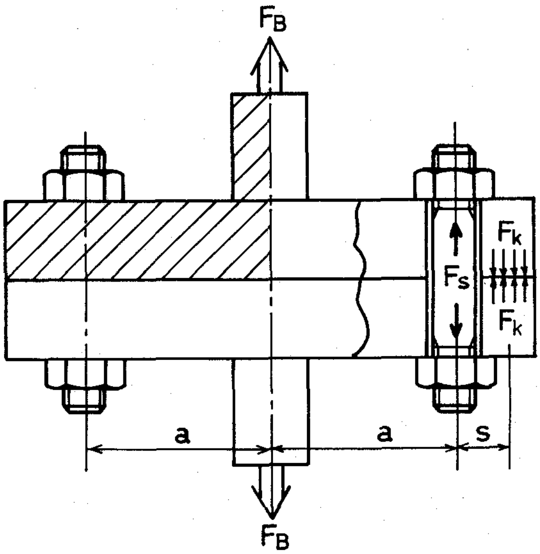

Fig. 1 T-flange bolted joint model

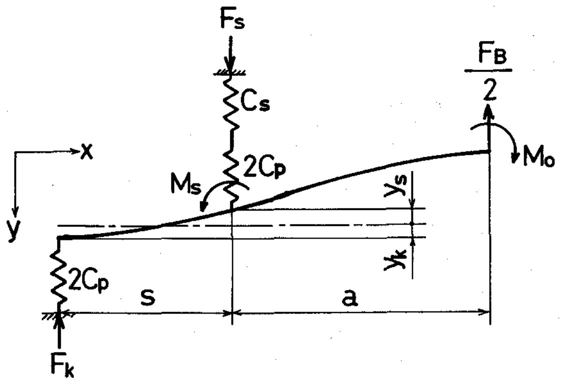

Fig. 2 Analytical model for T-flange bolted joint

reaction force concentrated, the angle of deflection is assumed to be zero.

The following symbols are used:

a[mm] : distance form the center of the bolt hole to the loading point
C_P[N/mm] : compliance of the flange
C_S[N/mm] : compliance of the bolt
F_B[N] : load
F_V[N] : preload of the bolt
F_k[N] : reaction force due to a lever action
F_S[N] : tension of the bolt
K_S[N-m/rad.] : bending compliance of the bolt
M_S[N-m] : bending moment acting on the bolt
s[mm] : distance from a point of the concentrated reaction force to the center of the bolt hole
y_k[mm] : shrinkage of the flange
y_S[mm] : elongation of the bolt

When the coordinate axes x and y are chosen as shown in Fig. 2, the equilibrium of the forces F_B, F_S, F_k is represented as

$$\frac{F_B}{2} - F_S + F_k = 0. \tag{1}$$

The curvatures of the elastic line are represented as follows:
For $0 < x < s$,

$$EJ_P \frac{d^2y}{dx^2} = -F_k x, \tag{2}$$

while for $s < x < s + a$,

$$EJ_P \frac{d^2y}{dx^2} = -F_k x + F_S(x - s) - M_S, \tag{3}$$

where

E[N/mm²] : Young's modulus of the flange
J_P[mm⁴] : geometrical moment of inertia of the flange.

The bending moment acting on the bolt is

$$M_S = K_S \frac{dy}{dx} \quad (x = s). \tag{4}$$

Bending compliance K_S of the bolt is represented as

$$K_S = \frac{E_b J_b}{L},$$

where

E_b[N/mm²] : Young's modulus of the bolt
J_b[mm⁴] : geometrical moment of inertia of the bolt
L[mm] : half of grip length.

The following boundary conditions are used:

(1) At $x=0$, $y=y_k=(F_k-F_v)/2C_p$, $dy/dx=0$,
(2) At $x=s$, $y=-y_s=-(F_s-F_v)/C_o$,
(3) At $x=s+a$, $dy/dx=0$,

where

$$C_o=\frac{2C_sC_p}{C_s+2C_p}.$$

The differential Eqs.(2) and (3) are integrated successively and then four equations are obtained with four integration constants. By substituting the boundary conditions into these equations, the following equations are reduced

$$F_k\left(\frac{K_sa}{EJ_p}+1\right)s^2+2F_kas+(F_k+F_s)a^2=0, \tag{5}$$

$$\frac{1}{EJ_p}\left[-\frac{F_ks^3}{6}+EJ_p\frac{(F_k-F_v)}{2C_p}\right]=-\frac{F_s-F_v}{C_o}. \tag{6}$$

By substituting the Eq.(1) into Eq.(6), a formula for the tension F_s of bolt is reduced as a function of distance s and load F_B as follows:

$$F_s=\frac{D_1}{D_2}\left(\frac{s^3}{12\,EJ_p}-\frac{1}{4C_p}\right)-\frac{D_3}{D_2} \tag{7}$$

where

$D_1=12C_pC_oEJ_pF_B$,
$D_2=-6EJ_p(2C_p+C_o)+2C_pC_os^3$,
$D_3=6EJ_p(2C_p+C_o)F_v$.

By substituting the Eq.(1) into Eq.(5), the load F_B acting on the T-flange bolted joint is obtained as a function of distance s as follows:

$$F_B=\frac{G_1F_v}{G_2+G_3+G_4+G_5}, \tag{8}$$

where

$G_1=6(C_o+2C_p)\{s^2(K_sa+EJ_p)+2EJ_pas\}$,
$G_2=-a^2C_pC_os^3$,
$G_3=6C_p(K_sa+EJ_p)s^2$,
$G_4=12EJ_pC_pas$,
$G_5=3EJ_pa^2(2C_p+C_o)$.

The bending moment M_s acting on the bolt is reduced from Eqs.(2), (3) and the boundary condition (1) as follows:

$$M_s = K_s \frac{F_k s^2}{2EJ_p}, \tag{9}$$

where each value of the compliances C_s of the bolt and C_p of the flange are calculated using a method described ref.3.

For a given preload, when the distance s is determined, the load F_B and the bending moment M_s are calculated by equations (8) and (9). And then the tension F_s of the bolt is determined using the equation (7).

The maximum distance s is from the center of the bolt hole to the edge of the flange because a point of the concentrated reaction can not positioned out of the flange.

3. Experiment

The T-flange bolted joint used for the present experiment is shown schematically in Fig. 3. The dimension for each part of the flange is shown in Fig. 4. As shown in this figure, a V-shape grooves on the contact surface of the flange are made at each bolt hole, since it is necessary to guide the leading wire of the strain gauge mounted on the bolt to outside of the T-flange bolted joint. The dimension of the two sizes of the bolts is shown in Fig. 5. As shown in this figure, two pairs of strain gauges were mounted on each bolt. The thread of both the bolts (a) and (b) is size of M12. The shank diameters of the bolts (a) and (b) are respectively 12 mm and 10 mm. The relation between load and stress in each bolt has been previously calibrated by using an universal testing machine. The

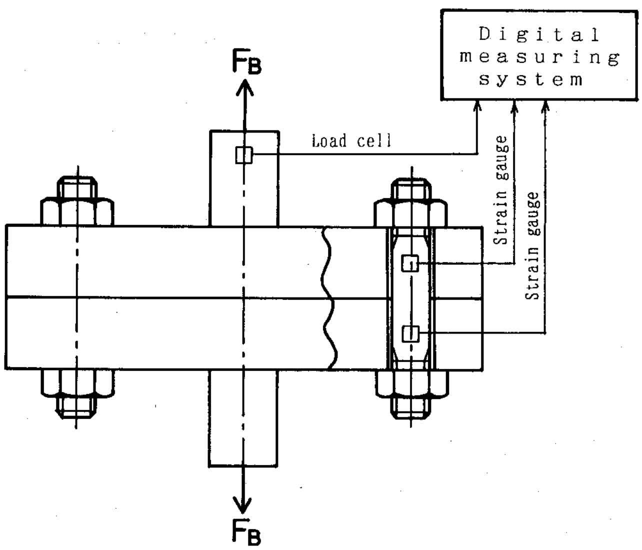

Fig. 3 Arrangement for measurement of load and stress

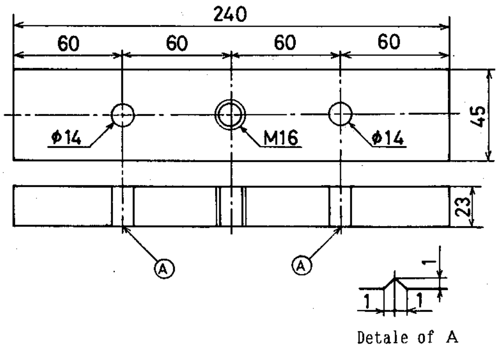

Fig. 4 Geometry of flange

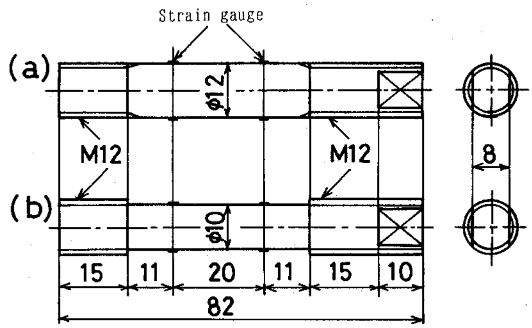

Fig. 5 Geometry of bolts

preload (18.1 kN or 29.4 kN) was applied to each bolt and was measured using the strain gauges which work a stress indicator, and then the T-flange bolted joint shown in fig. 3 was installed on the universal testing machine. Both the load acting on the T-flange bolted joint and the strain on the bolt were recorded using a digital static strain meter (UCAM-5A Kyowa Electric Instrument Co.).

4. Results and Discussions

Figures 6 to 9 show the relations between the load acting on the T-flange bolted joint and the additional axial stress or the additional bending stress in the bolt. In these figures, the experimental values are shown using symbols ○ and ●, and the values calculated under the same condition as the experiment are also shown by solid and dotted lines.

The calculated results shown in these figures indicate a higher increasing rate of stress than that measured by experiment. This may be caused by the fact that the distributed reaction force created by a lever action on the contact suface is assumed to be concentrated force and that the rigidity of the part of the thread engagement is ignored in this model.

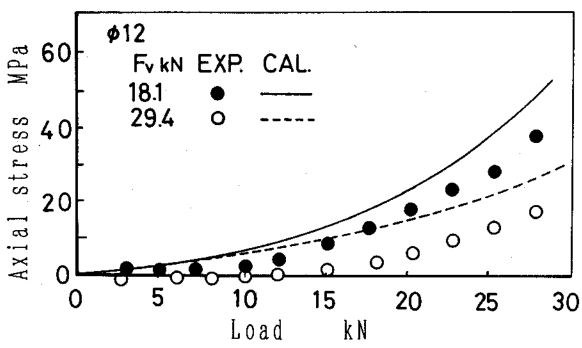

Fig. 6 Relation between load and additional axial stress

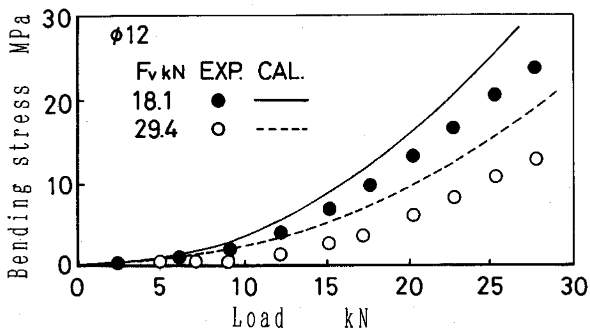

Fig. 7 Relation between load and additional bending stress

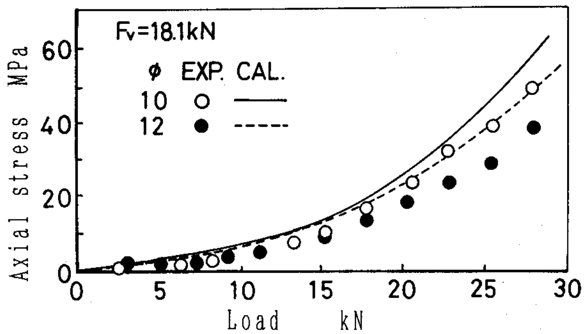

Fig. 8 Relation between load and additional axial stress

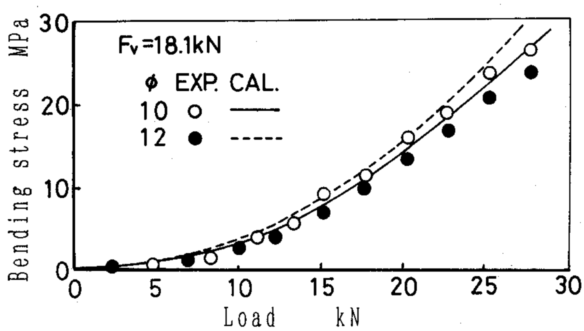

Fig. 9 Relation between load and additional bending stress

As shown in Figs. 6 and 7, the high preload of the bolt gives rise to the low increasing rate of the stress. This result has been understood generally so that plates clamped by low tightening force are separated by low external force each other. From the modeling analysis, however, this result is also explained by the fact that the point of the reaction

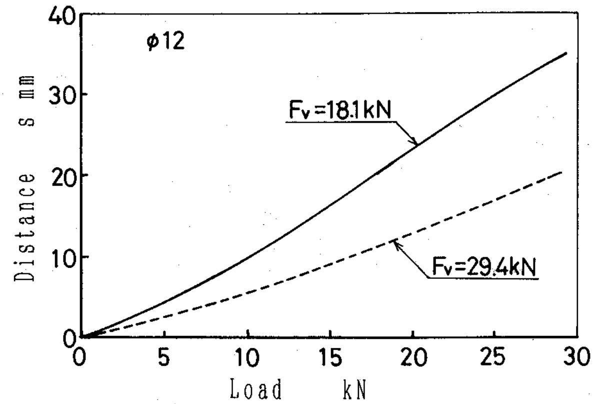

Fig. 10 Relation between load and distance from center of bolt hole to reaction force

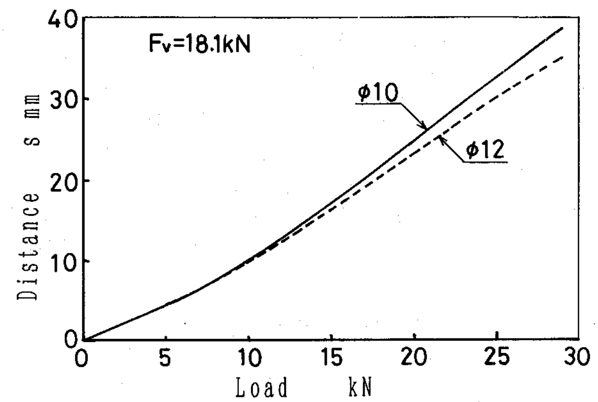

Fig. 11 Relation between load and distance from center of bolt hole to reaction force

force for the case of low preload moves faster to the direction of the flange edge than that for the case of high preload, as shown in Fig. 10.

As shown in Fig. 8, the small sized bolt results in the high increasing rate of the additional axial stress. The modeling analysis indicates that the point of the reaction force for the small sized bolt moves faster to the direction of the flange edge than that for the large sized bolt, as shown in Fig. 11. The present result shown in Fig. 8 is different from the well accepted idea. It has been claimed that the increasing rate of stress for small sized bolt is lower than that for large sized bolt. Further investigations may be necessary to understand this problem.

As shown in Fig. 9, the calculated value shows that the large sized bolt results in the high increasing rate of the additional bending stress, while the measured value shows that the small sized bolt results in the high increasing rate of that. This may be considered by the fact that the bending rigidity of the large sized bolt is taken to be larger than the actual bending rigidity of the bolt, since the rigidity of the part of the thread engagement is neglected and the bolt length is estimated by a half of grip length in the model.

Although the present model of the T-flange bolted joint has a simple structure which consists of springs and beam, the calculated results using this model are in good agreement with the experimental results, as shown in Fig. 6 to 9. In real cases, the bolted joint has various pairs of bearing surfaces. The frictional forces are created on the bearing surfaces of the bolted joint by slipping actions between the surfaces. Therefore, the various directions of the frictional force appear in the bolted joint. This may result in cancellation of the frictional forces in the bolted joint. The agreement between calculation and experiment may be obtained under such conditions described above.

5. Conclusion

The results may be summarized as follows:

(1) A simplified model for the T-flange bolted joint which consists of spring and beam is proposed.

(2) For taking into account the bending moment act on the bolt, a simple and useful method of the stress analysis in the bolt is proposed.

(3) The calculated results using this model agree considerably well with experimental results.

(4) For the axial stress, the present result is different from the well accepted idea that the increasing rate of stress for small sized bolt is higher than that for large sized bolt. Further investigations need to be developed on this problem.

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