Analysis of the Z term in the Variation of Latitude (2)

The Empirical Equation for the Variation of Latitude and the Determination of the Polar Coordinates

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緯度変化における Z 項の解析 (2)

緯度変化の実験式と極の決定法

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要 約

緯度変化に関しての問題に，極の座標を求めることと，緯度変化量 $\Delta\varphi$ の実験式をつくることとがある。

今緯度 φ，経度 λ の地の緯度変化量を $\Delta\varphi$ とし，極の座標を x，y とおけば，極の座標は次の式でとかれる。

$$\Delta\varphi = x\cos\lambda + \sin\lambda \qquad (1)$$

また $\Delta\varphi$ の実験式は，緯度変化曲線を描くことから，次の Fourier 級数で表わされる。

$$\left.\begin{aligned}\Delta\varphi = a_0 + a_1\cos\lambda + a_2\cos 2\lambda + \cdots + a_m\cos m\lambda \\ + b_1\sin\lambda + b_2\sin 2\lambda + \cdots + b_m\sin m\lambda\end{aligned}\right\} \qquad (2)$$

但し n は観測所の個数，$n \geqq 2m+1$ とする。

(1)，(2) 式を最小二乗法にかけてとき，その最確値を X，Y および，A_0，$A_1 \cdots\cdots$，A_m，B_1，B_2，$\cdots\cdots$，B_m とおけば，一般に

$$X \doteqdot A_1, \quad Y \doteqdot B_1 \qquad (3)$$

もしも観測所の経度 λ_j 緯度 φ_j を

$$\left.\begin{aligned}\lambda_j &= \lambda_o + (j-1)\frac{2\pi}{n} \\ \varphi_j &= \varphi_o \\ i &= 1,\ 2,\ \cdots,\ n\end{aligned}\right\} \qquad (4)$$

とおき，その地（これを理想観測地とよぶ）の緯度変化量 $\Delta\varphi_j$ を (2) 式によって推定し，その値 $\Delta\phi_j$ を求め，(1) 式に代入して，それを最小二乗法でとけば，次をうる

$$\left.\begin{aligned}X &= \frac{2}{n}\Sigma\,\Delta\phi_j\cos\lambda_j = A_1 \\ Y &= \frac{2}{n}\Sigma\,\Delta\phi_i\sin\lambda_j = B_1\end{aligned}\right\} \qquad (6)$$

すなわち，(2) の最適根の A_1，B_1 が極の座標 X，Y のよい推定値となった。

なお $m=1$ の時，(2) 式は木村式となる

$$\Delta\varphi = x\cos + y\sin\lambda + z \qquad (7)$$

$m=2$ の時は次のように表わされる。

$$\left.\begin{aligned}\Delta\varphi = x\cos\lambda + y\sin\lambda \\ + u\cos 2\lambda + v\sin 2\lambda + w\end{aligned}\right\} \qquad (8)$$

So far as the variation of latitude is concerned, we have the following problems.

First, we have to determine the coordinates of the pole.

Secondly, we have to find the empirical equation of $\Delta\varphi$.

Here, we should like to say that the latter is a means to reach the former. The coordinates of the pole can be solved by the following equation of the polar motion.

$$\Delta\varphi = x\cos\lambda + y\sin\lambda \qquad (1)$$

where X and Y are the coordinates of the pole, λ is the longitude, and $\Delta\varphi$ represents the variation of latitude.

The empirical equation is represented by FOURIER series, hence we have ;

$$\left.\begin{aligned}\Delta\varphi = a_0 + a_1\cos\lambda + a_2\cos 2\lambda \\ + \cdots + a_m\cos m\lambda \\ + b_1\sin\lambda + b_2\sin 2\lambda \\ + \cdots + b_m\sin m\lambda\end{aligned}\right\} \qquad (2)$$

where $n \geqq 2m+1$

If (1) and (2) are solved by the method of least squares, and if these most probable values are denoted by X, Y, $A_0, \cdots A_m$, and $B_1, \cdots B_m$, then we can generally say

$$X \doteqdot A_1, \qquad Y \doteqdot B_1$$

But if we solve (1) and (2) by the method of least squares at the ideal stations, we may have the following expressions.

$$X = A_1, \qquad Y = B_1. \qquad (3)$$

Here we want to demonstrate that expression (3) can be obtained by (1) and (2).

Now let λ_j and φ_j be the longitude and latitude of j-latitude observatory, where they are as follows

$$\left.\begin{aligned}\lambda_j &= \lambda_0 + (j-1)\frac{2\pi}{n} \\ \varphi_j &= \varphi_0 \qquad j = 1,\ 2,\ \cdots,\ n.\end{aligned}\right\} \qquad (4)$$

And let these stations be called the ideal ones. If $\Delta\varphi_j$ is the estimated value of $\Delta\varphi_j$, then (2) becomes

$$\left.\begin{aligned}\Delta\phi &= A_0 + A_1\cos\lambda_j \\ &+\cdots+ A_m\cos m\lambda_j \\ &+ B_1\sin\lambda_j \\ &+\cdots+ B_m\sin m\lambda_j\end{aligned}\right\}\qquad(5)$$

By solving (1) with respect to $\Delta\phi_j$ by the method of least squares, we can get the following results

$$\left.\begin{aligned}X &= \frac{2}{n}\,\Sigma\,\Delta\phi_j\cos\lambda_j = A_1 \\ Y &= \frac{2}{n}\,\Sigma\,\Delta\phi_j\sin\lambda_j = B_1,\end{aligned}\right\}\qquad(6)$$

thus we have got the most probable values as the coordinates of the pole by the solution of (2).

Obiter Dictum

If $m = 1$, in the equation (2), the equation may be as follows

$$\Delta\varphi = x\cos\lambda + y\sin\lambda + z \qquad (7)$$

where $n \geqq 3$.

This is called Kimura's equation.

And if $m = 2$, the equation (2) may be denoted by such an expression as

$$\begin{aligned}\Delta\varphi &= x\cos\lambda + y\sin\lambda \\ &+ u\cos 2\lambda + v\sin 2\lambda + w\end{aligned}\qquad(8)$$

where $n \geqq 5$.

The most probable X and Y obtained by solving (8) become the best estimated coordinates of the pole.

If, in the equation (8), $Z'$is expressed in the following way

$$Z' = u\cos 2\lambda + v\sin 2\lambda + w, \qquad (9)$$

then it corresponds to Kimura's term Z in (7), and also is the function of longitude. Therefore the variation of Z' is considered to be a function of λ.

In the equation (9), W is nonpolar variation. And we know it depends only upon the mean $\Delta\delta$ by which we mean here a variation of declination.

It is worthy of notice that the variation of Z' is useless in determining the coordinates of the pole.

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