Zonal Harmonic Analysis of Cosmic Ray Data from World-wide Network Cosmic Ray Stations : Method and its Application

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Abstract

The formulation to analyse three-dimensional cosmic ray anisotropy in interplanetary space has been given by Nagashima. In his paper, an emphasis was put on formulating the daily variation arising from a generalized anisotropy in space. This paper gives the details of analytical method how to derive the three-dimensional anisotropies of cosmic rays, that is, cosmic ray north-south asymmetry and pole-equator anisotropy in interplanetary space in reference to the Nagashima's formulation. In addition to it, some examples of its application to the practical analysis using cosmic ray data from the cosmic ray stations of worldwide network through the world-wide data center will be shown, illustrated and discussed.

1. Introduction

Cosmic ray intensities observed at stations at earth's surface have usually been resulted from a combined effect of various variations, which have different time and spatial variations. In general, it is very difficult to distinguish a single intensity variation from records obtained at a single station at the earth's surface. It is necessary that a close network of cosmic ray stations is set up widely at the earth's surface to overcome such a difficulty. Such a close network over the earth's surface has been set up since the International Geophysical Year (IGY) , using Simpson type counter, that is, IGY-type neutron monitor in the period of IGY and thereafter it has been replaced by the Carmichael's type neutron counter (super-neutron monitor) . Many stations have been closed for the period to the present since the IGY, whereas considerable numbers of station have come into operation newly during the period. Various methods to distinguish a single component of the variations from the cosmic ray data observed

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at the earth's surface have been proposed (e.g., Yoshida *et al.*, 1971 and 1973) .

The formulation to analyse three-dimensional cosmic ray anisotropy in interplanetary space has been developed in a new scheme and a frame work from an extensive analysis of the observed data has been provided by Nagashima (Nagashima, 1971, Nagashima *et al.*, 1971 and 1972) . The formulation gives a detailed method handling daily variation by solar anisotropy in which a method how to derive zonal harmonic components of cosmic rays in space is implied.

In this paper, we give the formulation for finding out the zonal harmonic components and the details of method of analysis for finding out the components which, especially, correspond to the north-south asymmetry and the pole-equator anisotropy respectively from cosmic ray data obtained at cosmic ray stations distributed over the earth's surface, in reference to the Nagashima's formulation (Nagashima, 1971) , together with some results of its practical analysis which are newly obtained from re-computation on the basis of somewhat different idea from earlier ones (Takahashi *et al.*, 1974, 1975, 1977, 1981 and 1983) .

2. Formulation

The formulation of cosmic ray daily variation arising from a generalized axis symmetric anisotropy in space has been developed by Nagashima (1971) as the basis of the extensive analysis which utilizes effectively all the associated harmonic components of daily variation. According to the formulation, the variational intensity distribution in space is defined by the following equation,

$$\delta J(P, \chi, \Lambda)/J(P)=F(\chi)G(P), \quad \cdots\cdots (2-1)$$

where P is the cosmic ray rigidity, $G(P)$ is the differential rigidity spectrum and $F(\chi)$ is called the space distribution. Quantity χ defines the cosmic ray incident direction (JO) relative to the reference axis (OR) as shown in Fig. 1.

Space distribution $F(\chi)$ in Eq. (2-1) may be expanded into a series of Legendre function as follows :

$$F(\chi)=\sum_{n=0}^{\infty} F_n(\chi)=\sum_{n=0}^{\infty} \eta_n P_n^0(\cos \chi), \qquad (2-2)$$

where $F_n(\chi)$ is called the n-th space distribution. Coefficient η_n is an arbitrary constant and called the magnitude of $F_n(\chi)$, where $|\eta_n|$ is called the absolute magnitude of $F_n(\chi)$. Since the n-th space distribution $F_n(\chi)$ in Eq. (2-2) does not always show the maximum in the direction of the axis OR (i.e. χ=0) shown in Fig. 1, the OR-axis is called the "reference axis of anisotropy" which is different from the "direction of anisotropy" hitherto used.

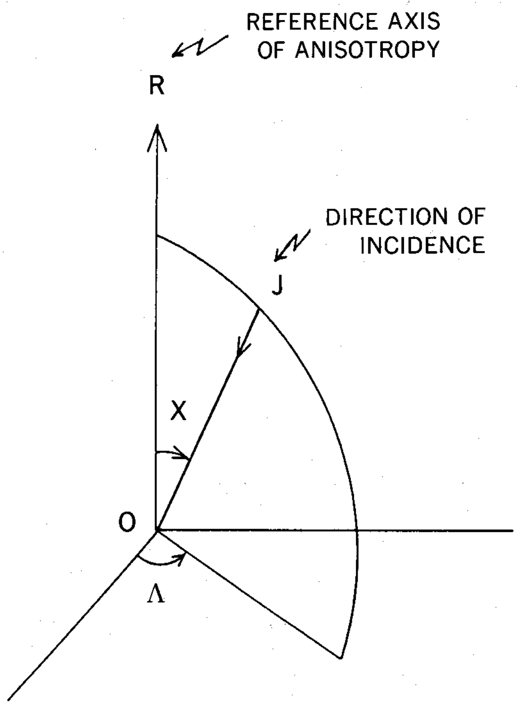

Fig. 1. A coordinate system illustrating the reference axis of anisotropy. Quantity χ defines the cosmic ray incident direction (JO) relative to the reference axis (OR). (after Nagashima, 1971)

If we make the transformation of the coordinate system shown in Fig. 1 into the equatorial coordinate system as is illustrated in Fig. 2, it follows by the addition theorem for spherical harmonics (Courant and Hilbert, 1943, or Morse and Feshbach, 1953) that

$$P_n^0(\cos \chi)=\sum_{m=0}^{\infty} P_n^m(\cos \theta_J)P_n^m(\cos \theta_R)\cos\{m(\alpha_J-\alpha_R)\}, \qquad (2-3)$$

where, from spherical trigonometry

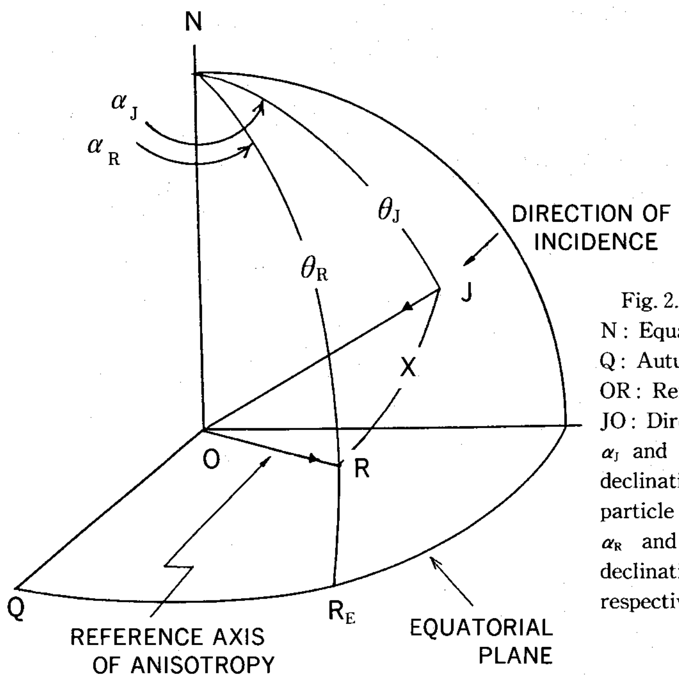

Fig. 2. Equatorial coordinate system.
N : Equatorial north pole
Q : Autumnal equinox.
OR : Reference axis of anisotropy
JO : Direction of incident particle
α_J and θ_J : Right ascension and co-declination of the direction of incident particle (OJ) , respectively
α_R and θ_R : Right ascension and co-declination of the reference axis (OR) , respectively (after Nagashima, 1971)

$$\cos \chi = \cos \theta_J \cos \theta_R + \sin \theta_J \sin \theta_R \cos(\alpha_J - \alpha_R) \quad \cdots\cdots (2-4)$$

in which α_R and θ_R are the right ascension and co-declination of the reference axis (OR) , respectively and α_J and θ_J are those of the incident direction JO, respectively as are shown in Fig. 2. The subscripts R and J stand for the directions of reference axis and of incident particle, respectively. Since the both functions, $P_n^m (\cos \theta_J)$ and $P_n^m (\cos \theta_R)$ in Eq. (2-3) are the semi-normalized spherical function by Schmidt (e.g., Chapman and Bartels, 1940) , if we denote them by $P_n^m (\cos \theta)$ in common, for brevity, $P_n^m (\cos \theta)$ may be given as follows:

$$P_{n,m} (\cos \theta) = P_n^m (\cos \theta) \qquad \text{when } m=0$$

and

$$P_n^m (\cos \theta) = 2 \left(\frac{(n-m)!}{(n+m)!} \right)^{\frac{1}{2}} P_{n,m} (\cos \theta) \qquad \text{when } m>0,$$

where the ordinary "associated Legendre function",

$$P_{n,m}(\mu)=(1-\mu^2)^{\frac{m}{2}}\frac{d^nP_n(\mu)}{d\mu^n} \qquad \mu=\cos\theta.$$

Consequently, it finds from Eq. (2-3) that Eq. (2-2) becomes

$$F(\chi)=\sum_{n=0}^{\infty}F_n(\chi)=\sum_{n=0}^{\infty}\{\sum_{m=0}^{n}f_n^m(\chi)\} \cdots\cdots (2-5)$$

and

$$F_n(\chi)=\sum_{m=0}^{n}f_n^m(\chi), \cdots\cdots (2-6)$$

where

$$f_n^m(\chi)=\eta_nP_n^m(\cos\theta_J)P_n^m(\cos\theta_R)\cos\{m(\alpha_J-\alpha_R)\} \quad \text{for } m=0,1,2,\cdots,n. \cdots (2-7)$$

The function $f_n^m(\chi)$ in Eq. (2-7) is called the projected component of space distribution. Especially, components f_n^m's assigned by a definite value of n are called the associate projected components of $F_n(\chi)$. The n-th space distribution produces $(n+1)$-terms of the associated component, which are independent each other. One of the components, that is, the one assigned by $m=0$ (i.e. f_1^0) does not produce any daily variation, owing to the absence of functional relation to $(\alpha_J-\alpha_R)$ (cf. Eq. (2-7)) . The north-south asymmetry of cosmic ray intensity belongs to this type of projected component arising from space distribution $F_n(\chi)$ and is expressed by f_1^0 . The term f_2^0 is similar to f_1^0 in character, corresponding to the pole-equator anisotropy which has been pointed out by some workers (Takahashi *et al.*, 1974, 1975 and 1977 ; Ely, 1977 ; Duggal and Pomerantz, 1978) .

The factor $P_n^m(\cos\theta_R)$ in Eq. (2-7) is called the "association factor" which defines the mutual relation among the associated projected components f_n^m's when co-declination of the reference axis is assigned by θ_R.

The factor $P_n^m(\cos\theta_J)$ in Eq. (2-7) may be called conventionally the "latitude distribution" of the projected component and expressed by $L_n^m(\theta_J)$ in order to distinguish itself from the space distribution. But, strictly speaking, a term of "declination distribution" should be used for this case rather than the "latitude distribution", although not familiar in the field of cosmic ray modulation. This is because the projected component is defined, fixed in space, not in the rotating system with the earth.

Solar anisotropy, defined by Eqs. (2-1), (2-5) and (2-7) produces an intensity (daily) variation $D(t)$, observed at a station on the earth, which may be given by a superposition of the intensity variations as follows:

$$D(t)=\sum_{m=0}^{\infty} D^m(t), \qquad (2-8)$$

where t is solar local time in hour.

The projected component $f_n^m(\chi)$ in Eqs. (2-6) and (2-7) also produces the following m-th harmonic component $D_n^m(t)$ of intensity (daily) variation at a station

$$D^m(t)=\sum_{n=m}^{\infty} D_n^m(t) \qquad (2-9)$$

It finds from the combination of Eqs. (2-8) and (2-9) that

$$D(t)=\sum_{m=0}^{\infty} D^m(t)=\sum_{m=0}^{\infty}\sum_{n=m}^{\infty} D_n^m(t) \qquad (2-10)$$

If we take the case $m=0$ when the spherical functions are called zonal surface harmonics, we have, from consideration of the relation between Eqs. (2-8), (2-9) and (2-10),

$$D_n^0(t)=\eta_n P_n^0(\cos\theta_R)c_n^0 \qquad (2-11)$$

and

$$c_n^0=(1/I)\int_{P_c}^{\infty} Y(P)G(P)L_n^0\{\theta_{OR}(P)\}\,dp, \qquad (2-12)$$

where

$$I=\int_{P_c}^{\infty} Y(P)dp \qquad (2-13)$$

and furthermore

$$L_n^0\{\theta_{OR}(P)\} = P_n^0\{\cos\theta_{OR}(P)\} \quad \cdots\cdots (2\text{-}14)$$

In the above equations, P_c, $Y(P)$ and I are the cosmic ray cut-off rigidity, the response function, and the average intensity of cosmic rays at the station respectively, and $L_n^0\{\theta_{OR}(P)\}$ is the declination (or latitude) distribution defined by Nagashima (1971), where denotes co-declination of cosmic ray asymptotic orbit at a station.

$D_n^0(t)$'s in Eq. (2-11) are the so-called n-th zonal harmonic components, which do not produce any daily variation. Therefore, hereafter $D_n^0(t)$'s will be designated simply by D_n^0's. D_0^0 means isotropic component of cosmic ray intensity and D_1^0 which is related to f_1^0 as mentioned before is observed as the north-south asymmetry of cosmic ray intensity and interpreted as the cosmic ray flow in the direction of the earth's rotation axis (Nagashima *et al.*, 1968). Higher order terms $D_n^0(t)$'s which have not observed yet, have complicated declination distributions respectively, as is known from Fig. 3. As is seen in Fig. 3, $P_2^0(\cos\theta_J)$, i.e. $L_2^0(\theta_J)$ is a north-south symmetric distribution, showing an anomaly at the equator, and is called the equatorial anomalous distribution

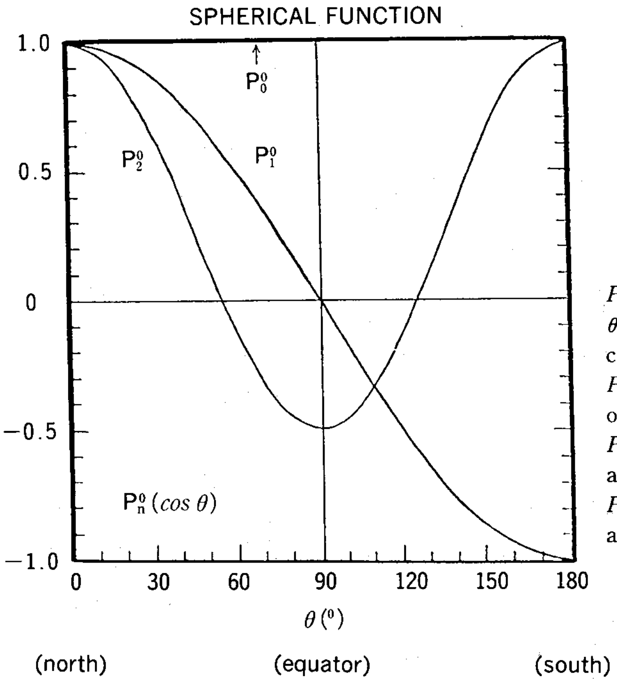

Fig. 3. Zonal harmonic functions $P_n^0(\cos\theta)$, for $n=0, 1, 2$, respectively. θ corresponds to the latitude; $\theta=90°$ corresponds to the equator. P_0^0 corresponds to the isotropic component of cosmic rays in space. P_1^0 corresponds to the north-south asymmetry of cosmic rays in space. P_2^0 corresponds to the pole-equator anisotropy of cosmic rays in space.

in contrast with the north-south asymmetry. Therefore we see that D_2^0 corresponds to the pole equator anisotropy.

The quantity c_n^0 is the coupling coefficient (or constant) between the zonal harmonic component D_n^0 and the projected component of the space distribution f_n^0 .

As is seen in Eq. (2-9) , the m-th harmonic component $D^m(t)$ is composed of a series of $D_n^m(t)$, which originates from the projected component f_n^m in Eq. (2-7) . Similarly to the definition of the components, $D_n^m(t)$'s assigned by a definite value of n are called the associated harmonic components of intensity (daily) variation, which can be produced by the n-th space distribution $F_n(x)$.

3. Method of Analysis

In Eq. (2-11) , for simplicity if we take the harmonic components up to the second order, that is, $n=0, 1, 2$, an intensity variation of cosmic rays at i-th station, I_i^{exp}, which is expected theoretically, may be given as follows:

$$\Delta I_i^{exp}=\sum_{n=0}^{2}\{\eta_n P_n^0(\cos\theta_R)\}\ c_{n,i}^0=a_0^0 c_{0,i}^0+a_1^0 c_{1,i}^0+a_2^0 c_{2,i}^0, \quad \cdots\cdots (3-1)$$

where

$$\left.\begin{aligned} a_0^0&=\eta_0 P_0^0(\cos\theta_R)\\ a_1^0&=\eta_1 P_1^0(\cos\theta_R)\\ a_2^0&=\eta_2 P_2^0(\cos\theta_R)\end{aligned}\right\} \quad \cdots\cdots (3-2)$$

If the intensity decrement of cosmic rays observed actually at the i-th station is denoted by ΔI_i^{obs} $(i=1, 2, 3, \ldots\ldots, j)$, the values of a_0^0, a_1^0 and a_2^0 in Eq. (3-2) are determined at every day (as function of UT) from the data of cosmic ray intensities at stations, $i=1, 2, 3, \ldots\ldots, j$, by the least square fitting method (see Appendix) , which is to minimize the following sum of square variations weighted by w's,

$$S=\sum_{i=1}^{j} w_i(\Delta I_i^{obs}-\Delta I_i^{exp})^2 \quad \cdots\cdots (3-3)$$

where w_i may be taken from the mean counting rates at the i-th station respectively.

The values of $c_{n,i}^0$ for almost all stations distributed over the earth's surface have been calculated from Eq. (2-12 and 2-13) by Yasue *et al.*, (1982) , using the following two types of the differential rigidity spectrum as $G(P)$ in the equation,

$$G(P)=\begin{cases} 0 & \text{for } P<P_L \\ \left(\frac{P}{10}\right)^{\gamma} & \text{for } P_L \leqq P \leqq P_H \\ 0 & \text{for } P>P_H \end{cases} \quad \cdots\cdots (3-4)$$

which is normalized at 10 GV (for brevity, this type of spectrum is referred to as "power type spectrum") and

$$G(P)=\left(\frac{P}{\gamma P_0}\right)^{\gamma} exp(\gamma-\frac{P}{P_0}) \quad \cdots\cdots (3-5)$$

which is also normalized at the rigidity γP_0, giving a peak intensity in the above spectrum for the positive γ (for brevity, this type of spectrum is referred to as "power-exponential spectrum") . P_L and P_H in the Eq. (3-4) are lower and higher cut-off rigidity in the power type spectrum, respectively and P_0 in Eq. (3-5) characterizes the decaying form in the higher rigidity side for the power-exponential type spectrum.

The values of S in Eq. (3-3) can be thus calculated by using the values of $c_{n,i}^0$ for the cases when the assumed spectrum form is either power type or power-exponential type, or the both types.

4. Discussion of the method and examples of the analysis

As is known from Eq. (2-11) , we can expect a great number of the zonal harmonic components corresponding to $n=0, 1, \ldots, \infty$, theoretically.

The case $n=0$ (i.e. D_0^0) corresponds to the isotropic component of cosmic rays in space. The case $n=1$ (i.e. D_1^0) corresponds to the cosmic ray north-south asymmetry. The case $n=2$ (i.e. D_2^0) corresponds to the pole-equator anisotropy. The higher terms

D_n^0's corresponding to $n \geqq 3$ have not been observed yet and moreover, any mechanism or phenomenon is expected neither theoretically nor experimentally. Even such a phenomenon corresponding to $n=2$ is difficult to detect by practical observation, as is shown by such a fact that we have only a few reports (Ely, 1977; Duggal and Pomerantz, 1978) till the present since the beginning of the cosmic ray observation. In other words, it may be said that, in general, such a phenomenon is unfamiliar with us, because it may be masked by the north-south asymmetry. If the change of cosmic ray intensity at polar region is taken place, that is, when the north-south asymmetry of the intensity is taken place due to such a change, the pole-equator anisotropy is regarded as being taken place, even in case with no intensity change at the equator region. To be detected, such a pole-equator anisotropy have to be distinguished clearly from the north-south asymmetry.

The method (Eq. (3-3)) given here may be useful to distinguish the pole-equator anisotropy from the north-south asymmetry and to find out the pole-equator anisotropy, independently on the north-south asymmetry.

Some examples of the analysis using Eq. (3-3) are shown below (Figs. 4 and 5). In these analyses we used $c^0_{0,i}$, $c^0_{1,i}$ and $c^0_{2,i}$ for each of which the following spectra are used for their evaluations from Eq. (2-12), respectively.

(i) power type spectrum with $\gamma=0$, -0.2, -0.5, and -1.0. For simplicity, each spectrum with such a γ-value designates their spectrum numbers as Nos. *1, 2, 3* and *4*. In other words, each of the numbers *1, 2, 3* and *4* corresponds to the power type spectrum with $\gamma=0$, -0.2, -0.5 and -1.0, respectively.

(ii) power-exponential type spectrum with the pairs ($\gamma=2.0$, $P=20$ GV), ($\gamma=2.0$, $P=30$ GV), ($\gamma=2.0$, $P=50$ GV), and ($\gamma=2.0$, $P=100$ GV). Similarly to the case of the power type spectrum, each spectrum designates as Nos. *5, 6, 7, 8* and *9*, respectively. Stating in more detail, for example, the power exponential spectrum with the pair ($\gamma=2.0$, $P=20$ GV) is called the "No. *5* spectrum" and similarly the others are called like that.

However, for $c^0_{0,i}$ in case of $n=0$, in Eqs. (2-11) and (3-3), only the power type spectrum (spectrum numbers *1, 2, 3* and *4*) is used, because such a type of spectrum is assumed to be plausible for the isotropic component. Thus, combination of these spectra whose number is ranging *1* to *9* amounts to $4\times 9\times 9=324$ cases. S-values are calculated on the basis of 27-day period which corresponds to each solar rotation

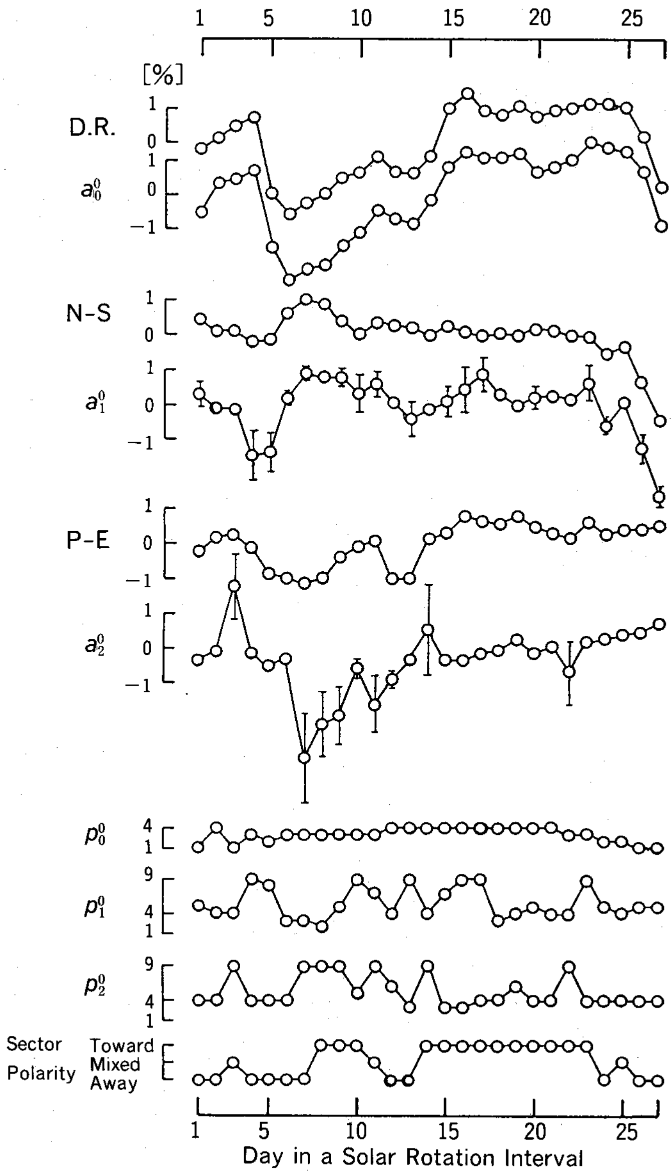

Fig. 4. Illustrating the results of analysis for the period, Solar Rotation No. 1873 (28 June-24 July, 1970).
Horizontal: days (from 1 to 27) during a solar rotation period (Rot. No. 1873)
Vertical: D. R. shows cosmic ray intensity in % at the cosmic ray station, Deep river (Height 145 *m*; Geograph. Lat. 46.10° N, Long. 77.50° W; Cut-off rigidity 1.41 GV, Count. Rate 19.35× $10^5/hr$). a_0^0, a_1^0 and a_2^0 correspond to 0-th, 1st and 2nd zonal harmonic components, respectively.
p_0^0, p_1^0 and p_2^0 show the best-fit differential rigidity spectrum corresponding to the 0-th, 1st and 2nd zonal harmonic components, respectively. Figures for p_0^0, p_1^0 and p_2^0 indicate the spectrum number ranging from 1 to 9.
N-S indicates the observed cosmic ray intensity difference between cosmic ray station at North region (Thule) and South region (McMurdo).
P-E indicates the cosmic ray intensity difference between polar regions: ((Thule-McMurdo)/2) and equatorial region (Huancayo) stations.

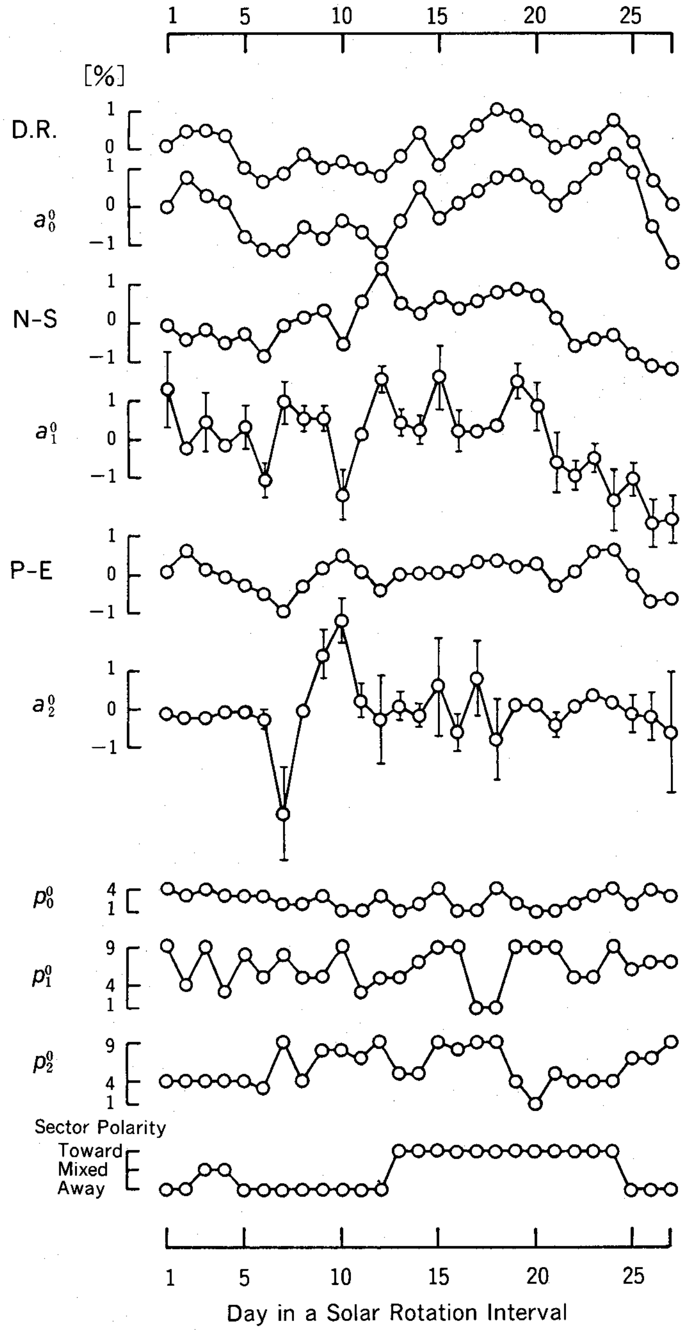

Fig. 5. The result of analysis of the cosmic ray data during the solar rotation period (Rot. No. 1841). Vertical and Horizontal axes are the same as those of Fig. 4.

interval for these cases by Eq. (3-3) , using the daily mean of the neutron intensities from the world-wide network cosmic ray stations through the world data center, *WDC -C2,* Itabashi, Tokyo. The cosmic ray intensity decrement, ΔI_i^{obs}, is taken as the difference between the mean intensity for the daily mean intensity for each day at each cosmic ray station during each solar rotation period and the corresponding 27-day interval to be analysed on *27*-day basis, as shown in Figs. 4 and 5. The number of the cosmic ray stations whose data are used in the analysis is about 44 in Fig. 4 and about 45 in Fig. 5 in the maximum, respectively. Such a low limit of the number of the stations in the analysis is mainly caused by lack of data at some stations, although the upper limit of the number of the stations was 65 at that time, as mentioned before.

It may be clear from Figs. 4 and 5 that the distinction among the isotropic component, the north-south asymmetry and the pole-equator anisotropy is relievable. Especially, it may be found that even the pole-equator anisotropy which is supposed to be difficult to detect in the first-hand data from the observing stations may be recognized clearly in contrast to the other components. From the other point of view it follows that the analysis method is useful and besides powerful to distinguish the pole -equator anisotropy from the north-south asymmetry and also from the isotropic component. As is convenient for a comparison with the corresponding quantities, that is, the neutron intensity at Deep River in contrast to the isotropic component a_0^0, N-S component from the corresponding cosmic ray stations in contrast to a_1^0, and P-E component from the corresponding cosmic ray stations (given as only a rough measure) in contrast to a_2^0, are shown in the figures (Figs. 4 and 5) . It might be noticed that there exists a good correlation between the Deep River neutron intensity and a_0^0 through the both figures (Figs. 4 and 5) . Such a tendency has been seen in the previous results (Takahashi *et al.*, 1974, 1975 and others) . An examination of the significance of the features (inclusive of a_1^0, a_2^0) will be left for the future.

5. Conclusion

From these, it may be concluded that this formulation and, accordingly, the method of analysis are plausible. At the same time, it finds that the pole-equator anisotropy phenomena as well as the north-south asymmetry ones have been frequently happened beyond our imagination. The critical examination and also a search of the mechanism of the results obtained will be a future problem.

Acknowledgements

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References

Courant, C. and D. Hilbert, Methoden Der Mathematischen Physik I , 2 Aufl., 1943.
Chapman, S., and J. Bartels, Geomagnetism, Vol. II, London Press, Oxford, 1940.
Duggal, S. P. and M. A. Pomerantz, Geophys. Res. Letters, 8, 625, 1978.
Ely John, T. A., J. Geophys. Res., 82, 3463, 1977.
Morse, M. and H. Feshbach, Methods of Theoretical Physics, Pt. II, McGraw-Hill Book Company, Inc., 1953.
Nagashima, K., S. P. Duggal and M. A. Pomerantz, Planet. Sci., 16, 29, 1968.
Nagashima, K., Rep. Ionos. Space Res. Japan, 25, 189, 1971.
Nagashima, K. and H. Ueno, Rep. Ionos. Space Res. Japan, 25, 212, 1971.
Nagashima, K., H. Ueno, K. Fujimoto, Z. Fujii and I. Kondo, Rep. Ionos. Space Res. Japan, 26, 1, 1972.
Nagashima, K., H. Ueno, K. Fujimoto, Z. Fujii and I. Kondo, ibid., 26, 31, 1972.
Takahashi, H., N. Yahagi and K. Nagashima, Proc. Int. Symp. on STP, São Paulo, 1, 431, 1974.
Takahashi, H., N. Yahagi and K. Nagashima, Proc. 14th Int. Cosmic Ray Conf., München, West-Germany, 4, 1236, 1975.
Takahashi, H., N. Yahagi and K. Nagashima, Proc. 15th Int. Cosmic Ray Conf., Plovdiv, Bulgaria, 3, 284, 1977.
Takahashi, H., N. Yahagi and K. Nagashima, Proc. 16th Int. Cosmic Ray Conf., Kyoto, Japan, 3,

1979.
Takahashi, H., N. Yahagi and K. Nagashima, Proc. 17th Int. Cosmic Ray Conf., Paris, France, 10, 197, 1981.
Takahashi, H., N. Yahagi and K. Nagashima, Proc. 18th Int. Cosmic Ray Conf., Bangalore, India, 10, 148, 1983.
Yasue, S., S. Mori, S. Sakakibara and K. Nagashima, Rep. Cosmic Ray Res. Lab., Vol. 7, 1982, Cosmic Ray Res. Lab., Nagoya Univ., Nagoya, Japan.
Yoshida, S. and S-I. Akasofu, J. Geophys. Res., 76, 1, 1971.
Yoshida, S., N. Ogita, S-I. Akasofu and L. G. Gleeson, J. Geophys. Res., 78, 6409, 1973.

Appendix

Details of the least square fitting method used in this analysis

It is given in the section 3 in the body of this paper that the zonal harmonic components of cosmic rays in space are obtainable by minimizing the following sum (Eq. 3-3)

$$S=\sum_{i=1}^{j} w_i(\Delta I_i^{obs}-\Delta I_i^{exp})^2$$

by the least square fitting method. This is the purpose to give the details of the least square fitting method used in the practical analyses.

In the following, the notations a and w are the same ones as in the body of the paper, whereas f_n^0 may be corresponded to $\Delta I^{\exp}$, ϕ may be corresponded to c_n^0, and x may be corresponded to i. In other words, if the station number i is designated as a variable, 'function' which is given as a numerical value may be corresponded to the coupling coefficient c.

A function $f(x)$ of variable x may be expanded as follows:

$$f(x)=a_0\phi_0(x)+a_1\phi_1(x)+a_2\phi_2(x) \quad \cdots\cdots (A-1)$$

where x is an independent variable, ϕ is a function of x (e.g. trigonometric series or spherical functions), and a is a constant to be determined by the method of least squares to be mentioned below.

If an observed value is designated by y and also its theoretical value from the calculation is designated by f, the residual, v, may be represented by

$$v=y-f \qquad \cdots\cdots (A-2)$$

Determination of a is attributed to find the most probable value of a which satisfies the following relation

$$S=[v^2]=minimum \qquad \cdots\cdots (A-3)$$

by means of the least square method, where

$$[v^2] \equiv [v \cdot v] \equiv \sum_i (v_i \cdot v_i)$$

The symbol [ ] means summation of the quantities and will be used throughout this section in place of Σ.

Expressing the above relations in more detail,

$$v(x, y ; a_j) \equiv y-f(x) \qquad j=0, 1, 2.$$
$$S(x, y ; a_j) \equiv [v^2] = [(y-f)^2]$$

The necessary condition for which S takes its minimum value is

$$\frac{\partial S}{\partial a_j}=0. \qquad \cdots\cdots (A-4)$$

Namely

$$\frac{\partial S}{\partial a_j}=\frac{\partial}{\partial a_j}\{\Sigma(v^2)\}=2\Sigma\{v\frac{\partial}{\partial a_j}(y-f)\}=0$$

Consequently

$$[\phi_j \cdot f] = [\phi_j \cdot y] \qquad j=0, 1, 2. \cdots\cdots (A-5)$$

Expressing Eq. A-5 in more detail

$$\left[\begin{pmatrix}\varphi_0\\ \varphi_1\\ \varphi_2\end{pmatrix}\cdot\left\{(\varphi_0,\ \varphi_1,\ \varphi_2)\cdot\begin{pmatrix}a_0\\ a_1\\ a_2\end{pmatrix}\right\}\right]=\left[\begin{pmatrix}\varphi_0\\ \varphi_1\\ \varphi_2\end{pmatrix}\cdot y\right]$$

and furthermore

$$\left[\begin{pmatrix}\varphi_0\varphi_0 & \varphi_0\varphi_1 & \varphi_0\varphi_2\\ \varphi_1\varphi_0 & \varphi_1\varphi_1 & \varphi_1\varphi_2\\ \varphi_2\varphi_0 & \varphi_2\varphi_1 & \varphi_2\varphi_2\end{pmatrix}\cdot\begin{pmatrix}a_0\\ a_1\\ a_2\end{pmatrix}\right]=\left[\begin{pmatrix}\varphi_0\ y\\ \varphi_1\ y\\ \varphi_2\ y\end{pmatrix}\right].$$

Therefore

$$\begin{pmatrix}[\varphi_0\varphi_0] & [\varphi_0\varphi_1] & [\varphi_0\varphi_2]\\ [\varphi_1\varphi_0] & [\varphi_1\varphi_1] & [\varphi_1\varphi_2]\\ [\varphi_2\varphi_0] & [\varphi_2\varphi_1] & [\varphi_2\varphi_2]\end{pmatrix}\cdot\begin{pmatrix}a_0\\ a_1\\ a_2\end{pmatrix}=\begin{pmatrix}[\varphi_0\ y]\\ [\varphi_1\ y]\\ [\varphi_2\ y]\end{pmatrix}\ . \quad \cdots\cdots\cdots (A-6)$$

Since the observed data are not always equi-weighted ones, the "weight" for v^2 should be taken into consideration. Then Eq. A-4 may be replaced by the following

$$\frac{\partial}{\partial a_j}\{\Sigma(w\cdot v^2)\}=0 \quad \cdots\cdots\cdots (A-7)$$

and furthermore

$$[w\cdot\phi_j\cdot f] = [w\cdot\phi_j\cdot y]\ , \qquad j=0,\ 1,\ 2.$$

namely

$$\begin{pmatrix}[w\varphi_0\varphi_0] & [w\varphi_0\varphi_1] & [w\varphi_0\varphi_2]\\ [w\varphi_1\varphi_0] & [w\varphi_1\varphi_1] & [w\varphi_1\varphi_2]\\ [w\varphi_2\varphi_0] & [w\varphi_2\varphi_1] & [w\varphi_2\varphi_2]\end{pmatrix}=\begin{pmatrix}[w\varphi_0\ y]\\ [w\varphi_1\ y]\\ [w\varphi_2\ y]\end{pmatrix}\ . \quad \cdots\cdots\cdots (A-8)$$

The practical analyses by this method were done on the basis of 27-day interval which corresponds to a solar rotation period. y is taken as a deviation in % of the cosmic ray intensity in daily mean at each station at each day for the 27-day interval from the mean value of cosmic ray intensities at each station during the corresponding solar rotation interval. w is determined from the counting rate at each cosmic ray station which is divided by a proper number (e.g., 10^3) under such consideration that the data do not become less precise. Since the coupling coefficient c is dimensionless, a is naturally evaluated in unit of %. The station number i is ranging from 1 to 65, which is dependent of the date when the cosmic ray data are obtained at the stations. At some date, some stations may provide their data, but some other stations may lack their data at the same date. For such a reason, the data from about 45 stations were, on an average, available for each day during each solar rotation period throughout the course of the practical analyses in this study.