ECONOMIC THEORY OF LIFE PROCESS: ON LIFE AND LABOR POWER AS JOINT PRODUCTS

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1 Accomplishment of Classical Wage Theory by E. Engel

The purpose of this paper is to investigate the economic value of life of human being. The characteristic of our methods is to treat both life and labor power as joint products of a life process. We shall show how the life span is given from the economic point of view, in what case the value of labor power is positive and finally what rolls the mode of life plays. Our basic stand point is the production cost theory of wages which is explored by classical economists, e. g. A. Smith, D. Ricardo. In short, we can say that our analysis is to show the logical consequence of that theory by using modern methods of theoretical economics.

The classical economists defined the wages from the view point of production costs of labor, which is mainly composed of three parts. The first part is the amount of value of living goods for the laborer. The second part is the cost of reproduction of the laborer, in other words, the expense of bringing up his children. The third part is the cost of education for the laborer to have a average level of labor skill. Unfortunately, they stopped to develop the theory further. It is probably because they thought it is unnecessary.

The economist who broke through the limitation of classical economist, metamorphosed the wage theory into a life theory and established the economic theory of life on a classical form is Ernst Engel (1821–1896). He is famous as a statistician who discovered the Engel's Law on consumption expenditure. However,

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his other side as a economist who analyze the life process has been mostly forgotten. His theories are in the "Der Preis der Arbeit" (1866)[1] and the "Der Werth des Memschen" (1888)[2]. These works are basically affected by Smith as said by himself. Then the key concept which connect the two is a mechanistic view of life of human being.[3]

In the "Der Preis der Arbeit", Engel divides a span of life into three parts, young periods, labor periods and old periods. Then he shows lists of costs for each period to be included in labor cost. The expense during young periods, which we call the initial value of life, is considered as what should be repaid at the second labor periods. Especially, we are interested in the Engel's treatment of insurance. In his work, the insurance prepared for cases that the depreciation of the initial value of life is not completed by some accidents, is included in the cost of labor. Engel calculates the cost of labor actually in the "Der Preis der Arbeit". It is evolved in the "Der werth des Menschen". In the latter work, he introduces a new cost, the value of new life. Then he calculates the total value of a laborer at a beginning labor period with the sum of he cost of living goods and the value of new life. However, the value of new life given by him is rather arbitrary. He does not indicate conditions which ensure the positive value of new life, and the quantity of the value.

This Engel's stand point is totally different from that of classical economists, who thought that population tends to over supply relative to its necessity for production. Thus the new life is something like free goods and value less. This concept is expressed in the "An Essay on the Principle of Population" by Malthus extensively. Therefore, the positive value of new life is the second notable achievement by Engel. We shall pose a complete specification on this problem in the latter part of this paper.

2 Natural Life Span and Effective Life Span of Labor Power

Here we start our discussions with showing a consistent system of life process.

1) Engel [1].
2) Engel [2].
3) This point of view is stated in Smith [12], p. 101.

We can pose a question for Engel's way to calculate the cost of labor power. In his works, the Life span of labor power is previously given as a average span. There is no doubt that the life of human being has his own span of life which is cnstrained by his physiological condditions. Thus we define the natural life span as the average of the maximum life span for some periods. It is clear that the natural life span (NLS) is not equal to the length which human being can continue to provide his labor power. It is another theoretical problem to be solved. Thus Engel's way is meaningful only as a first approximation. We shall define the effcctive life span of labor powes (ELS) as the span which human being can continue to provide his labor power effectively. In this chapter, we shall show how ELS will be given.

Now we introduce basic notations. $d_k(i)$'s ($k=1, 2, \cdots, n$; $i=0, 1, 2, \cdots, m$) are the quantity of k-th goods consumed by one unit of the i-th generation, e. g. one thousand people of that generation, where n is the number of goods and m is NLS. Then we have to consider the valuation system to estimate these goods. We may evaluate them with the production price system or the labor value system. When we adopt the former system and it is denoted p_k, $k=1, 2, \cdots, n$, then the total value of living goods $D(i)=p_1d_1(i)+p_2d_2(i)+, \cdots, +p_nd_n(i)$. On the other hand, when the latter system is denoted as s_k, $k=1, 2, \cdots, n$, then $D(i)=s_1d_1(i)+s_2d_2(i)+, \cdots, +n_nd_n(i)$.[4] In this paper, the difference of the valuation system does not cause any significant problems. In other words, since we focus on the production cost of labor service, when we use the former system, we regard it as "wage", and when we use the latter system, regard it as "value of labor power" or "necessary labor" defined by Marx. For this reason, we use the labor value system in the same way as classical economist.

On the other hand, we have to assume that the value configuration of goods is previously given. This assumption is not so weak. Because, in general, the

4) S. Mitsuchi, in [5], threw light on the significance of the life process based on a evolution of classical analysis and asserted to change our view point from "the reproduction of labor" to "the reproduction of life" in our economic analysis. The stand point is completely affirmative for us. The paper also focuses on the investigation how we treat service labor. Thus it is better for our model to include this type of direct labor in the value of living goods. To introduce it, however, will bring no change of our result when we adopt the labor value system. The quantity of labor supply can be regarded as the net supply when we employ the price system of production cost. Then there is no difference in results.

value system of goods and the value of labor service is determined simultaneously. Certainly in the classical system of labor value (there exists no joint production and no alternative proces of production), we can separate the two system completely. Apart from this point, we cannot preserve such type of simplicity. However, since modern theoretical economics has been simplifying the life process, the assumption must be permitted to introduce. It is a kind of simplification for the purpose of the acquisition of new knowledges about our capitalist economy.

We make all young periods, as defined by Engel, shrink to the 0-th generation. Thus the 0-th generation is unique generation which do nots supply labor service. On the other hand, we make no difference for the other generations, like labor generations and old generation as done by Engel.

$e(i)$, $(i=1, 2, \cdots, m; 0<e(i)\leqq 1)$ means probability that one unit of the (i-1)-th generation can maintain their lives to the i-th generation. It is not subjective probability. It should be define depending on observations of statistical facts. $l(i)$, $(i=1, 2, \cdots, m)$ is quantity of labor power (or labor service) supplied by one unit of the i-th generation, which is measured by hour. We assume $l(i)>0$ for all i. Because, though it may be rational that older generation cannot have labor power, it leads to trivial solutions of our problems posed in the following part of this paper. Furthermore, it is unrealistic to $l(i)=0$ for medium generations. However, if we remove this assumption, all of our propositions stated in three sections of this chapter hold. $t(i)$, $(i=1, 2, \cdots, m)$ is the value of life at the beginning of that period. Until section 4, $t(0)$, i. e. the value of new life, is previously given as 0. Finally, v is value of labor service measured by labor, i. e. value of of labor power.

Now we construct the valuation system of life process. As specified by Engel, the cost of labor service is composed from three elements, living goods, depreciation and insurance. Each cost of the i-th generation is written with our notation as follows,

cost of living goods $=D(i)$,

cost of depreciation $=t(i)-t(i+1)$,

cost of insurance $=t(i+1)[1-e(i+1)]$,

cost of labor service $=vl(i)$.

Thus, we have,

$$D(i)+[t(i)-t(i+1)]+t(i+1)[1-e(i+1)]=vl(i).$$

Finally,

$$t(i)+D(i)=vl(i)+t(i+1)e(i+1). \quad (i=1,\ 2,\ \cdots,\ m-1) \tag{1}$$

If i=0 for (1), since t(0)=0 and 1(0)=0, we have,

$$D(0)=t(1)e(1). \tag{2}$$

Also, for i=m, since e(m+1)=0,

$$t(m)+D(m)=vl(m). \tag{3}$$

The equations (1), (2) and (3) express a value system of life process. This system is a kind of the production price system with fixed equipment. Therefore, as well known, there may be some negative values in t(i)'s, which are nonsense economically. To avoid this problem, we can pose a planning problem as follows.

$$\max.\ v$$

s. t.

$$D(0)\geqq t(1)e(1)$$

$$t(i)+D(i)\geqq vl(i)+t(i+1)e(i+1) \quad (i=1,\ 2,\ \cdots,\ m-1)$$

$$t(m)+D(m)\geqq vl(m)$$

$$v,\ t(i)\geqq 0 \quad i=1,\ 2,\ \cdots,\ m$$

It is clear that the negative value problem can be completely solved in this system. There may be, however, some readers who have questions that the life processes are not controlled by any people, thus this system is seriously unrealistic. Certainly, we do not know the organizer of life processes. We must, however, pay attention to the fact that the above system is a sub-system. Now we suppose that D(i)'s are valued by production prices. Then the sub-system is consistent with the valuation system to introduce the wage-profit frontier for the von Neumann techology, in which the general profit rate is minimized under the constraint of processes[5]. On the other hand, when they are valued by labor values, the above system is consistent with the generalized labor value system which is posed by Morishima.[6]

Then the dual problem is represented as follows.

5) See Morishima [6], [7].
6) See Morishima [8], [9], [10].

$$\min. \sum_{i=0}^{m} D(i)y(i)$$

s. t.

$$e(i)y(i-1) \geqq y(i) \quad (i=1, 2, \cdots, m)$$

$$\sum_{i=1}^{m} l(i)y(i) \leqq 1$$

$$y(i) \geqq 0 \quad (i=0, 1, 2, \cdots, m),$$

where y(i)'s are the population of i-th generation. The first and second inequality show that lives which survive to the next generation are not necessarily become the object of labor power. The third inequality is a constraint that at least one unit of labor power must be produced.[7] In the followıng, the latter problem is called as "Main problem" or (M1) simply and the former as "dual problem, (m1)".

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7) Our specifications of life process are based on the assumption of a stationary state. However, if the population of labor power is growing for some exogenous reasons, we may have to change the system. In order to examine this proble m, let $y_t(i)$ be the y(0) in a period t, and normalize as $y_t(0)=1$. Assume that the rate of population growth is constant at g. Then t(1) on the period t+1 is e(1), that is, $y_{t+1}(1)=e(1)$ and $y_{t+2}(2)=e(1)e(2)$. Since y(0) is growing at the rate g, $y_{t+1}(0)=1+g$. By repeating the same procedure, we have the following table.

$y_t(0)=1$		
$y_{t+1}(0)=1+g$	$y_{t+1}(1)=e(1)$	
$y_{t+2}(0)=(1+g)^2$	$y_{t+2}(1)=e(1)(1+g)$	$y_{t+2}(2)=e(1)e(2)$
$y_{t+3}(0)=(1+g)^3$	$y_{t+3}(1)=e(1)(1+g)^2$	$y_{t+3}(3)=e(1)e(2)(1+g)$
....	....	....
....	....	....

In general, we can rewrite the constraints of generations as follows.

$e(1)y_t(0)=y_{t+1}(1)$

$e(2)y_t(1)=y_{t+1}(2)$.

Thus the steady growth at the rate g is represented as follows.

$e(1)y(0)=(1+g)y(1)$

$e(2)y(1)=(1+g)y(2)$.

The proportion of generations at a period is,

$y(0) : y(1) : y(2)=(1+g)^2 : (1+g)e(1) : e(1)e(2)$.

This result is the same as we obtained by the above table. Therefore it seems necessary to consider the population growth. Even so, we do not have to change our system. Because we can think that the probability of survival e(i)'s imply the growth factor. Thus, let e*(i)'s be the original probability and we can regard the probability e(i)'s as follows.

$$e(i)=\frac{e^*(i)}{1+g}$$

Consequently it is unnecessary to consider the population growth in our course of discussion.

Moreover, the above discussions are also applicable on the interest rate which is neglected in our systems.

In stead of the assumption that all D(i)'s are positive, let us employ the following assumption throughout this paper.

Assumption 1

Necessary labor is positive.

In other words, for every feasible solution of (M1) the value of objective function is positive. It is clear that this assumption is weaker than that every D(i) is positive.

Then we define ELS and QELS strictly.

Definition 1

Effective Life Span (ELS) is the shortest life span which attains the minimum necessary labor, and Quasi Effective Life Span (QELS) is the longest life span which also attain it.

QELS shall play important rolls in the later section on life styles.

Now let us assume that ELS is until j-th generation. We examine what configuration of inequalities and equalities holds for the restrictions of (M1) the solutions (y(0), y(1), y(2), ⋯, y(m))'s. Owing to our definition of ELS, there exists a solution which $y(i)>0$ for $i=0, 1, 2, \cdots, j$ and $y(i)=0$ for $i=j+1, j+2, \cdots, m$. Then what relations of inequalities do they have on the restrictions of (M1)? Is every inequality to j-th one related with equality, i. e. $e(i)y(i-1)=y(i)$ for $i=1, 2, \cdots, j$? If some of them do not hold with equality, the simplicity of our problem shall be lost. The next lemma, however, show us that such situations never occur, in other words, the scales of ELS generations satisfy equalities.

Lemma 1

Let us assume that one of solutions satisfies the restriction of (M1) as follows.

$$e(i)y(i-1)=y(i) \quad (i=1, 2, \cdots, s)$$

$$e(s+1)y(s)>y(s+1)$$

$$e(i)y(i-1)\geqq y(i) \quad (i=s+2, \cdots, m)$$

$$\sum_{i=1}^{m} l(i)y(i)=1,$$

where s does not mean ELS and $y(i)>0$ for $i=0, 1, \cdots, s$, $y(i)\geqq 0$ for $i=s+1, \cdots, m$. Then $y^*(i)$ $(i=0, 1, 2, \cdots, m)$ which satisfies the following restriction is also the solution of (M1).

$$e(i)y^*(i-1)=y^*(i) \quad (i=1, 2, \cdots, s)$$

$$\sum_{i=1}^{s} l(i)y^*(i)=1$$

$$y^*(i)>0 \quad (i=1, 2, \cdots, s)$$

$$y^*(i)=0 \quad (i=s+1, \cdots, m).$$

Proof

Let Y be a vector of solution, i. e. $Y=(y(0), y(1), y(2), \cdots, y(m))$. Also consider vector $Y^i=(y^i(0), y^i(1), \cdots, y^i(i), 0, 0, \cdots, 0)$ for $i=s, s+1, \cdots, m$, where the dimension is the same as Y and each element satisfies the following conditions.

$$e(k)y^i(k-1)=y^i(k) \quad (k=1, 2, \cdots, i)$$

$$\sum_{k=1}^{i} l(k)y^i(k)=1.$$

It is clear that we can construct them uniquely. Now consider to express Y by means of Y^i's. It is important to notice that Y and each Y^i are equivalent for the proportion of elements from the first up to the s-th, though they are different for the scale. Furthermore each Y^i is linearly independent. Therefore we can express Y by Y^i's, in other words, using α^i $(i=s, s+1, \cdots, m)$,

$$Y=\alpha^s Y^s+\alpha^{s+1}Y^{s+1}+, \cdots, +\alpha^m Y^m. \tag{5}$$

Then we can show $\alpha^i \geqq 0$ for $i=s, s+1, \cdots, m$. Let us assume $\alpha^k<0$ for some k. For k-th and k+1-th elements of the above equation,

$$y(k)=\alpha^k y^k(k)+\alpha^{k+1}y^{k+1}(k)+, \cdots, +\alpha^m y^m(k)$$

$$y(k+1)=\alpha^{k+1}y^{k+1}(k+1)+, \cdots, \alpha^m y^m(k+1).$$

Owing to our premise of the lemma,

$$e(k+1)y(k) \geqq y(k+1).$$

Since $e(k+1)y^i(k)y^i(k+1)$ for $i+k+1, k+2, \cdots, m$, we have,

$$\alpha^k Y^k(k) \geqq 0.$$

This and $y^k(k)>0$ contradict $\alpha^k<0$. Thus $\alpha^i \geqq 0$ for $i=s, s+1, \cdots, m$.

Let $L=(0, i(1), i(2), \cdots, l(m))$ and $D=(D(0), D(1), D(2), \cdots, D(m))$. Define inner product with '・'. Since $Y \cdot L=1$ and $Y^i \cdot L=1$ for $i=s, s+1, \cdots, m$,

$$\alpha^s+\alpha^{s+1}+, \cdots, +\alpha^m=1. \tag{6}$$

On the other hand, $\alpha^s>0$, because if $\alpha^s=0$ then $e(s+1)y(s)>y(s+1)$ does not hold. Since each Y^i is one of feasible solution of (M1), for the objective function, they cannot make it strictly smaller than the optimal solution Y. Thus $Y \cdot D \leqq Y^i \cdot D$ for i=s, s+1, ⋯, m. If $Y \cdot D < Y^s \cdot D$, from (6) and α^s this leads to $Y \cdot D < Y^s \cdot D + Y^{s+1} \cdot D +, \cdots, + Y^m \cdot D$. This contradicts (5). Therefore $Y \cdot D = Y^s \cdot D$. This means that Y^s is one of optimal solutions. (Q.E.D)

We see from this lemma that the configuration of ELS generations are uniquely decided by their probability of survival. Let ELS be to j-th generation. Then the fact that there exists a strict inequality in conditions $e(i)y(i-1) \geqq y(i)$ $i=1, 2, \cdots, j$ contradicts that ELS is up to j-th generation. If for some $k(<j)$ $e(k+1)y(k) > y(K+1)$ then from this lemma there exists a solution as $y(i)=0$ $i=k+1, k+2, \cdots, j$. Thus we can preserve the simplicity of our problem.

3 Labor Conditions and Value of Labor Power

Let us start our discussions on the relation between the main problem and the dual problem. We use the duality theorem[8)], which states that the minimized necessarly abor as a solution of (M1) and the value of labor power as that of (m1) are identical, in other words, for the solutions v and y(i)'s,

$$\sum_{i=0}^{m} D(i)y(i) = v. \qquad (1)$$

The requirements of the theorem that the two problem have to have feasible solutions are clearly satisfied under our premise. Since the left hand side of (1) is positive owing to Assumption 1, the right hand side, the value of labor power, is also positive. Multiply each solution of (M1), y(i)'s, by each inequalities of (m1) formed by solutions v and t(i)'s respectively, and adding up for each side, we have,

$$\sum_{i=1}^{m} t(i)y(i) + \sum_{i=0}^{m} D(i)y(i) \geqq v \sum_{i=1}^{m} l(i)y(i) + \sum_{i=1}^{m} y(i)e(i)y(i-1). \qquad (2)$$

On the other hand, multiply each solution of (m1), t(i)'s and v, by each inequalities of (M1) formed by solution y(i)'s respectively, and adding up, we have,

8) See, for example, Nikaido [6].

$$v\sum_{i=1}^{m} l(i)y(i)+\sum_{i=1}^{m} t(i)e(i)y(i-1)\geqq\sum_{i=1}^{m} t(i)y(i)+v. \qquad (3)$$

Taken into consideration of (1), we see that the left hand side of (2) is equivalent to the right hand side. Moreover the right hand side of (2) and the left hand side of (3) is identical. Thus we see that every side for these two equations is completely equivalent. It means from (2) that if some solution of (m1) brings strict inequality to constraint equation for i-th generation, $y(i)$ for every solution of (M1) has to be zero, that is, if $t(i)+D(i)>vl(i)+t(i+1)e(i+1)$ then $y(i)=0$ for all solutions. Similarly, from (3), if $e(i)y(i-1)>y(i)$ for some solution then $t(i)=0$ for all solutions. Thus as mentioned in last section, it is impossible that $\sum l(i)y(i)\geqq 1$ hold with strict inequality, because of $v>0$.

These preparations easily lead us to the next lemma which shall hereafter play important roles.

Lemma 2

The ELS generations have the positive value of life.

Proof

Let ELS be up to j-th generation. First we see that there is no generation within ELS which cannot retain its input value to the outpt. In other words, if for some $k(k\leqq j)$,

$$t(k)+D(k)>vl(k)+t(k+1)e(k+1),$$

then $y(k)=0$ as to the solution of (M1). It means k-th generation does not belong to ELS. It is contradiction. Thus if $t(k)=0$ $(k\leqq j)$, we have following equation.

$$D(0)=t(1)e(1)$$

$$t(i)+D(1)=vl(i)+t(i+1)e(i+1) \quad (i=1,\ 2,\ \cdots,\ k-2)$$

$$t(k-1)+D(k-1)=vl(k-1) \quad (\text{as } t(k)=0).$$

The maximum value of labor power v is decided by this system. On the other hand, consider the following equation system.

$$e(i)y(i-1)=y(i) \quad (i=1,\ 2,\ \cdots,\ k-1)$$

$$\sum_{i=1}^{k-1} l(i)y(i)=1$$

y(i)'s satisfying these equalities are uniquely given. It is easily seen that, for above two systems,

$$\sum_{i=0}^{k-1} D(i)y(i)=v.$$

Since from the duality theorem v is equivalent to the minimum necessary labor, this means that it is attained by more shorter life span than ELS. It contradict our definition of ELS. (Q.E.D)

Next we investigate the given conditions for each life process. It is expressed by a composition of coefficients, (D(i), l(i), e(i+1))'s. As defined in the last section, D(i) is nothing but the amount of value of living goods. First, let us pay attention to l(i). l(i) means the quantity of labor power expended during some period. Thus it coincides with Marx's concept of "working-day" in the Capital. The extension of working day, in other words, the increase in l(i) is the most general expression of the intensification of labor. Then it shall cause the deterioration of laborer and the decrease in the probability of death of the following generations, in other words, the decrease in e(k), k=i+1, i+2, …,.

In the following part, we shall focus on the change of l(i)'s, e(i)'s and influences that it brings to the value of labor power or the necessary labor. The first result is next theorem.

Theorem 2

The intensification of labor on any ELS generation decrease the value of labor power.

Proof

Let us assume that ELS is up to J-th generation and l(k) $(k \leqq j)$ change to l'(k) as $l(k) < l'(k)$. The solution for the former situation is expressed by (v, t(i), i=1, 2, …, m) and for the latter is by (v', t'(i), i=1, 2, …, m). Paying attention to the sign of inequality, we can easily find the fact that the solution for the latter also satisfies the conditions for the former system. It show that the latter solution is one of the feasible solution of the former system. Since the feasible solution has not to exceed the maximum solution on the value of labor power, $v \geqq v'$.

On the other hand, as previously mentioned, t(j+1)=0. Thus v is decided by the following system.

$$D(0)=t(1)e(1)$$

$$t(i)+D(i)=vl(i)+t(i+1)e(i+1) \quad i=1, 2, \cdots, j\text{-}1$$

$t(j)+D(j)=vl(j)$.

Then the solution after the change satisfies the following inequalities.

$$D(0)\geqq t'(1)e(1)$$
$$t'(i)+D(i)\geqq v'l(i)+t'(i+1)e(i+1) \quad i=1, 2, \cdots, k-1$$
$$t'(k)+D(k)\geqq v'l'(k)+t'(k+1)e(k+1)$$
$$t'(i)+D(i)\geqq v'l(i)+t'(i+1)e(i+1) \quad i=k+1, k+2, \cdots, j,$$

where the following equations are omitted. Subtract each equation of the former system from the corresponding equation of the latter system respectively. Then,

$$0\geqq (t'(1)-t(1))e(1)$$
$$t'(i)-t(i)\geqq (v'-v)l(i)+(t'(i+1)-t(i+1))e(i+1) \quad i=1, 2, \cdots, k-1$$
$$t'(k)-t(k)\geqq v'l'(k)-vl(k)+(t'(k+1)-t(k+1))e(k+1)$$
$$t'(i)-t(i)\geqq (v'-v)l(i)+(t'(i+1)-t(i+1))e(i+1) \quad i=k+1, k+2, \cdots,$$
$$t'(j)-t(j)\geqq (v'-v)l(j)+t'(j+1).$$

Let us assume $v'=v$. From the first equation, since $e(1)>0$, $t'(1)-t(1)\leqq 0$. From the second equation, since this and $v'-v=0$, we see $t'(2)-t(2)\leqq 0$. By the same way, we have $t'(k)-t(k)\leqq 0$. Moreover, in the last equation, since $t'(j+1)\geqq 0$ and $v'-v=0$, $t'(j)-t(j)\geqq 0$. Repeating it upward, we have $t'(k+1)-t(k+1)\geqq 0$. On the other hand, $v'l'(k)-vl(k)>0$ because of $v'=v\geqq 0$ and $l'>l$. Thus in k-th equation, the left hand side is nonpositive, on the other hand, the right hand side is positive. It contradicts the direction of inequality. Thus finally we have $v>v'$. (Q.E.D)

Next we shall prove the theorem on the effect of the change of $e(i)$.

Theorem 3

The decrease of the survival probability of ELS generation increases the value of labor power.

Proof

Let ELS be up to j-th generation and the valuation system be v, $t(i)$ $(i=1, 2, \cdots, m)$. We investigate the effect which is caused by a change that $e(k)$ the survival probability of $k-1$・th generation involved in ELS decreases and reaches to $e'(k)$. Let ELS in the latter situation be up to s-th generation and the valuation system be v', $t'(i)$ $(i=1, 2, \cdots, m)$.

First, we can easily find out $v'\geqq v$. This is because the solution composed by v, $t(i)$'s is a feasible solution of the problem with $e'(k)$. In other words, if we apply the solution to the latter system, we have the following inequality,

$$t(k-1)+D(k-1)>vl(k-1)>vl(k-1)+t(k)e'(k),$$

and the situation of inequality of the other equations is the same as the former system. It means that v, t(i)'s satisfy the restrict condition of the problem with e'(k). v in the feasible solution cannot be larger than maximum solution, i. e., $v' \geqq v$.

Next we show that $s<j$ and $k \leqq s+1$ do not hold simultaneously. In the former system, which is before changing, the value preservation equations from the s+1·th to the j-th are expressed as follows.

$$t(i)+D(i)=vl(i)+t(i+1)e(i+1) \quad (i=s+1, \cdots, j-1)$$

$$t(j)+D(j)=vl(j) \quad (t(j+1)=0).$$

Since $k \leqq s+1$, the above equations do not involve e(k). After changing of e(k), the system is altered as follows.

$$D(s+1) \leqq v'l(s+1)+t'(s+2)e(s+2) \quad (t'(s+1)=0)$$

$$t'(i)+D(i) \leqq v'l(i)+t'(i+1)e(i+1) \qquad (i=s+2, \cdots, j).$$

Subtracting the latter equations from the former equations respectively, we have,

$$t(s+1) \leqq (v-v')l(s+1)+(t(s+2)-t'(s+2))e(s+2)$$

$$t(i)-t'(i) \leqq (v-v')l(i)+(t(i+1)-t'(i+1))e(i+1) \quad (i=s+2, \cdots, j-1)$$

$$t(j)-t'(j) \leqq (v-v')l(j)-t'\ (j+1)e(j+1).$$

In the first equation, since $t(s+1)>0$ and $v-v' \leqq 0$, we see $t(s+2)-t'(s+2)>0$. By this fact, in the second equation, we see $t(s+3)-t'(s+3)>0$. Repeating the procedure, finally we have $t(j)-t'(j)>0$. On the other hand, In the last equation, this result contradicts the fact that the right hand side nonpositive. Thus $s<j$ and $k \leqq s+1$ do not hold simultaneously.

Then we investigate the other cases.

(α) The case of $s<j$ and $s+1<k$.

Subtracting each equation after changing from each equation before chaging, we have the following equations.

$$0=(t(1)-t'(1))e(1)$$

$$t(i)-t'(i)=(v-v')l(i)+(t(i+1)-t'(i+1))e(i+1) \quad (i=1, \cdots, s-1)$$

$$t(s)-t'(s)=(v-v')l(s)+t(s+1).$$

Assume $v=v'$. Investigating from the first equation, we easily find out $t(s)-$

$t'(s)=0$. On the other hands, since $s<j$, $t(s+1)>0$. These facts cause a contradiction in the last equation. Thus $v<v'$.

(β) The case of $s=j$.

By the same procedure as the above case, we have,

$$0=(t(1)-t'(1))e(1)$$

$$t(i)-t'(i)=(v-v')l(i)+(t(i+1)-t'(i+1))e(i+1) \quad (i=1, \cdots, k-2)$$

$$t(k-1)-t'(k-1)=(v-v')l(k-1)+t(k)e(k)-t'(k)e'(k)$$

$$t(i)-t'(i)=(v-v')l(i)+(t(i+1)-t'(i+1))e(i+1) \quad (i=k, \cdots, j-1)$$

$$t(j)-t'(j)=(v-v')l(j)+t(j+1)e(j+1).$$

Assume $v=v'$. Investigating from the first equation, we find out $t(k-1)=t'(k-1)$. On the other hand, investigating from the last equation, we have $t(k)=t'(k)$. Since $e(k)>e'(k)$ and $t(k)>0$, $t(k)e(k)>t'(k)e'(k)$. It contradicts the sign of equality for the k-th equation. Thus $v<v'$.

(γ) The case of $s>j$.

Subtract each equation after changing from each equation before changing, we can show equations as follows.

$$-t'(j+1)\geqq(v-v')l(j+1)+(t(j+2)-t'(j+2))e(j+2)$$

$$t(i)-t'(i)\geqq(v-v')l(i)+(t(i+1))e(i+1) \quad (i=j+2, \cdots, s-1)$$

$$t(s)-t'(s)\geqq(v-v')l(s)+t(s+1)e(s+1).$$

Assume $v=v'$. Since, in the first equation, $t'(j+1)>0$, we have $t(j+2)-t'(j+2)<0$. Repeating it, finally $t(s)-t'(s)<0$. Since theright hand side of the last equation is nonnegative, it causes contradiction. Thus $v<v'$.

Thus in all cases, we see that $v<v'$ has to hold. (Q.E.D.)

These two theorems express a antinomy proposition(9). If the intensification of labor is performed to the i-th generation, then it directly causes the decrease of the value of labor power by Theorem 2. On the other hand, it tends to reduce the probability of survival to the generation after that. Theorem 3 says it causes the increase in the value of labor power. Thus in capitalist economies, the increase in the death probability owing to the intensification of labor has to be controlled so as not to increase the value of labor power consequently. In order to endure the intensification of labor, the formation of the strong body or

9) This proposition are intensively investigated by Marx. For example, see Marx [4], p. 266, p. 269.

the improvement of health of laborer is indispensable for capitalist economies. Because it prevent from the increase of death probability.

4. Procreation and Value of New Life

So far we have been expanding our discussion under a important restriction, that is, new lives are supplied so sufficiently that the value of them is always zero. We shall remove it in this section. Let us introduce two kind of new notations. First let c(i) (i=1, 2, ⋯, m) be the amount of new lives which is produced through the life process of one unit of i-th generation. Assume, naturally, at least one of them is strictly positive. Second let t(0) be the value of new life. Then the reproduction condition of life is expressed as follows.

$$\sum_{i=1}^{m} c(i)y(i) \geqq y(0) \qquad (1)$$

$$e(i)y(i-1) \geqq y(i),\ y(i) \geqq 0,\ (i=1,\ 2,\ \cdots, m).$$

The basic assumption is,

Assumption 2

Life has the ability to maintain the number of individual.

It means that the above system has a nontrivial solution. We call this situation that the necessary procreation condition is satisfied. Furthermore the necessary life span of procreation (NLSP) is defined as follows.

Definition 2

The length of life which is necessary to maintain the number of individual by procreation is called the necessary life span of procreation.

The following lemma is a generalized version of the above result. We show it without proof.

Lemma 3

The system of inequalities,

$$\sum_{i=1}^{k} c(i)y(i) \geqq y(0)$$

$$e(i)y(i-1) \geqq y(i),\ y(i) \geqq 0,\ (i=1,\ 2,\ \cdots,\ k)$$

has a nontrivial solution, if and only if,

$$e(1)c(1) + e(1)e(2)c(2) + ,\cdots, + e(1)e(2) \cdots e(k)c(k) \geqq 1, \qquad (2)$$

is satisfied.

Since the right hand side of (2) is a monotonously increasing function for k, NLSP is up to k-th feneration as to the minimum k satisfying (2).

Now we are in a position to reconstruct a more generalized main problem as follows.

$$\min. \sum_{i=1}^{m} D(i)y(i)$$

s. t.

$$\sum_{i=1}^{m} c(i)y(i) \geqq y(0)$$

$$e(i)y(i-1) \geqq y(i), \ y(i) \geqq 0, \quad (i=1, 2, \cdots, m) \qquad \text{(M2)}$$

$$\sum_{i=1}^{m} l(i)y(i) \geqq 1$$

Similarly, we specify the dual problem.

$$\max. \ v$$

s. t.

$$t(0)+D(0) \geqq t(1)e(1)$$

$$t(i)+D(i) \geqq vl(i)+t(i+1)e(i+1)+t(0)c(i) \quad (i=1, 2, \cdots, m-1) \qquad \text{(m2)}$$

$$t(m)+D(m) \geqq vl(m)+t(0)c(m)$$

$$v, \ t(i) \geqq 0 (i=1, 2, \cdots, m)$$

We can show, under assumption 2, that both of the two problem have solutions and the optimal values of objective function coincide each other. Since, under assumption 2, the main problem has a feasible solution and the value of the objective solution is bounded below, the dual problem (m2) has a optimal solution. In order to show that the maximum solution of the (m2) is bounded above, the following lemma is indispensable.

Lemma 4

Let t be a column vector of n dimension, d be a column vector of m dimension and A a $n \times m$ matrix. If a set $T=\{t \mid d \geqq At, \ t \geqq 0, \ d \geqq 0\}$ is not bounded, then the nonnegative and nonzero vector t exists and $0 \geqq At$.

The proof of this lemma is in Washida [13].

Now we can show that the value of labor power satisfying conditions of (m2) is bounded above. Let us suppose the reverse situation. Then from the above lemma, there exists some v, t(i) i=0, 1, 2, ⋯, m, which satisfy

$$t(0) \geqq t(1)e(1)$$
$$t(i) \geqq vl(i)+t(i+1)e(i+1)+t(0)c(i) \quad i=1, 2, \cdots, m-1$$
$$t(m) \geqq vl(m)+t(0)c(m)$$

Clearly we can chose as $v>0$. Thus

$$t(0) \geqq t(1)e(1)$$
$$t(i) \geqq t(i+1)e(i+1)+t(0)c(i) \quad i=1, 2, \cdots, m-1$$
$$t(m) \geqq t(0)c(m),$$

and since we assume at least one l(i) is strictly positive, at least one equation hold with the strict sign of inequality. Then it is clear we can chose t(i)'s as follows.

$$t(0)>t(1)e(1)$$
$$t(i)>t(i+1)e(i+1)+t(0)c(i) \quad i=1, 2, \cdots, m-1$$
$$t(m)>t(0)c(m).$$

This inequalities, however, contradicts assumption 2. Because if the system (1) has a solution, the above system does not have a nonnegative solution. It can be showed immediately from the Gale's famous Alternative Theorem[10]. Thus v is bounded above. Moreover it is clear that (m) has a feasible solution. Hence both (M2) and (m2) have optimal solu+ions.

Now we are in a position to investigate meanings to have introduced new life into the system explicitly. It is, first of all, to show conditions that the value of new life is strictly positive. For that purpose, we first prove the following lemma.

Lemma 5

Let QELS be up to j-th generation. Then for every solution of (m1) in last section, which does not procreation explictly, the following inequality is hold.

$$D(j+1)>vl(j+1)+t(j+2)e(j+2). \qquad (3)$$

Proof

Suppose that (3) hold with the sign of equality for a solution. Then some natural number $k(m \geqq k \geqq j+1)$ exists and we have the following equations.

10) See Gale [3].

$$D(j+1)=vl(j+1)+t(j+2)e(j+2)$$

$$t(i)+D(i)=vl(i)+t(i+1)e(i+1) \quad i=j+2, \cdots, k-1 \qquad (4)$$

$$t(k)+D(k)=vl(k)$$

$$t(i)>0, \quad i=j+2, \cdots, k.$$

In other words, the preservation equations of value for generation whose value of life is positive hold with equality. We can show it as follows. Suppose that in spite of $t(j+2), \cdots, t(k)>0$, for some i

$$t(i)+D(i)>vl(i)+t(i+1)e(i+1) \quad j+2 \leqq i \leqq k$$

is hold. Clearly, i=j contradicts the hypothesis. When $i>j+1$, we can slightly reduce t(i) with maintaining v and the sign of inequality. Then the $i-1$·th equation holds with inequality. Thus we can slightly reduce $t(i-2)$. Repeating the procedure, we finally get $D(j+1)>vl(j+1)+t(j+2)e(j+2)$. It contradicts our hypothesis. Thus (4) is true.

Now let $Y'=(y'(0), y'(1), \cdots, y'(k), 0, 0, \cdots, 0)$ be a feasible solution of (M1) without procreation, satisfying

$$e(i)y'(i-1)=y'(i) \quad i=1, 2, \cdots, k$$

$$\sum_{i=1}^{k} l(i)y'(i)=1 \qquad (5)$$

On the other hand, $Y=(y(0), y(1), \cdots, y(j), 0, 0, \cdots, 0)$, satisfying

$$e(i)y(i-1)=y(i) \quad i=1, 2, \cdots, j$$

$$\sum_{i=1}^{j} l(i)y'(i)=1$$

is a optimal solution of (M1). It can be easily inferred from the proof of lemma 1. Since optimal solutions with longer life span than it do not exist and $k>j$, we obtain

$$v=\sum_{i=1}^{j} D(i)y(i)<\sum_{i=1}^{k} D(i)y'(i). \qquad (6)$$

On the other hand, multiplying each constraint equation of (ml) which has the sign of equality up to the k-th generation and consist of optimal solution (v, t(1), $\cdots$, t(m)) by each element of Y', we have

$$\sum_{i=1}^{k} t(i)(y'(i)-e(i)y'(i-1))+\sum_{i=0}^{k} D(i)y'(i)=v\sum_{i=1}^{k} l(i)y'(i).$$

From (5), we can reform this equation as follows.

$$\sum_{i=0}^{k} D(i)y'(i)=v.$$

It contradicts (6). (Q.E.D.)

Now we are in a position to provide the final result of this section.

Theorem 3

When NLSP is shorter than QELS, the value of new life is zero. When NLSP is longer than QELS, it is positive.

Proof

Let QELS be up to j-th generation and NLSP be up to k-th generation.

First we shall prove the former proposition. Thus $k<j$. Owing to our definition of QELS, a vector $Y'=(y'(0), y'(1), \cdots, y'(j), 0, 0, \cdots, 0)$ satisfying

$$e(i)y'(i-1)=y'(i) \quad i=1, 2, \cdots, j \tag{7}$$

$$\sum_{i=1}^{j} l(i)y(i)=1$$

is a optimal solution. On the other hand, from $k<j$ we obtain

$$e(1)c(1)+e(1)e(2)c(2)+, \cdots, +e(1)e(2)\cdots e(j)c(j)>1.$$

By this and (7), we have

$$\sum_{i=1}^{j} c(i)y'(i)>y'(0) \tag{8}$$

Thus Y' is a feasible solution of (M2). Moreover since (M2) is, simply, a problem added one restrict equation to (M1), the necessary labor of (M2) is equal to or more than that of (M1). Consequently we see that Y' is a optimal solution. Owing to the characteristics of dual problem, (8) means $t(0)=0$ for the optimal solution of (m2).

The latter proposition can be proved as follows. Let $v', t'(1), t'(2), \cdots, t'(m)$ be the solution of (m1). They satisfy the following equations.

$$\begin{aligned} &D(0)=t'(1)e(1) \\ &t'(i)+D(i)=v'l(i)+t'(i+1)e(i+1) \quad i=1, 2, \cdots, j-1 \\ &t'(j)+D(j)=v'l(j) \\ &D(j+1)>v'l(j+1)+t'(j+2)e(j+2) \\ &t'(i)+D(i)\geqq v'l(i)+t'(i+1)e(i+1) \quad i=j+2, \cdots, m, \end{aligned} \tag{9}$$

where the strict inequality for $j+1$·th generation is proved by lemma 5. On the other hand, since $k>j$ and the scales of generations at least up to k-th in (M2) have to be positive, some $k'(m \geqq k' \geqq k > j)$ exists and, for the solution of (M2), $y(0), y(1), \cdots, y(k) > 0, y(k+1) = y(k+2) =, \cdots, = y(m) = 0$. Then the equations of (m2) up to k'-th generation are connected with the sign of equality. Moreover $t(k+1)=0$ because of $e(k+1)y(k)>y(k+1)$. Thus we have

$$t(0)+D(0)=t(1)$$
$$t(i)+D(i)=vl(i)+t(i+1)e(i+1)e(i+1)+t(0)c(i) \quad i=1, 2, \cdots, k'-1 \quad (10)$$
$$t(k')+D(k')=vl(k')+t(0)c(k').$$

First we can confirm that the solution of (m1) is one of feasible solution of (m2) with $t(0)=0$. Since the value of labor power of the feasible solution has not to be larger than that of the optimal solution, we obtain

$$v \geqq v'. \quad (11)$$

Next, subtracting each equation of (10) from each equation of (9), we have

$$t(0)=(t(1)-t'(1))e(1)$$
$$t(i)-t'(i)=(v-v')1(i)+(t(i+1)-t'(i+1))e(i+1)+t(0)c(i) \quad i=1, 2, \cdots, j-1$$
$$t(j)-t'(j)=(v-v')1(j)+t(j+1)e(j+1)+t(0)c(j)$$
$$t(j+1)<(v-v')1(j+1)+(t(j+2)-t'(j+2))e(j+2)+t(0)c(j+1)$$
$$t(i)-t'(i) \leqq (v-v')1(i)+(t(i+1)-t'(i+1))e(i+1)+t(0)c(i) \quad i=j+2, \cdots, k'-1$$
$$t(k')-t'(k') \leqq (v-v')1(k')-t'(k'+1)e(k'+1)+t(0)c(k').$$

Suppose $t(0)=0$. Then moreover, suppose $v=v'$. On the last equasion, we see $t(k')-t'(k') \leqq 0$ because of $y'(k') \geqq 0$. Then on the $k'-1$·th equation, we have $t(k'-1)-t'(k'-1) \leqq 0$. Repeating it, we finally get $t(j+1)<0$. It contradicts the nonnegative constraint of $t(j+1)$. Thus $v \neq v'$ when $t(0)=0$. By this and (11) we see $v>v'$. On the other hand, the solution $v, t(0)=0, t(1), \cdots, t(m)$ of (m2) is one of the solution of (m1). Thus $v \leqq v'$. It is a contradiction. Therefore $(0)>0$. (Q.E.D)

Owing to this theorem, we see what situation makes the value of new life positive or zero. The criterion consist of the comparison between NLSP and QELS. NLSP represnts the power of procreation. When NLSP is relatively short the ability of procreation is weak. Then it may lead to the positive value of new life. Malthusian perspective is diferent from it. They saw the strong tendency toward excess population and the valueless new life. Classical economists are not so different from them.

Now, it is necessary to remark on the actuality of the positive value of new life. Since it is a special case that ELS is different from QELS, we will discuss about NLSP and ELS. First, Though it is related to the situation of economic growth, it cannot be thought that NLSP is over forty years old. On the other hand, we can approximately regards the average age of retirement as the ELS. In such situation, since NLSP is shorter than ELS, the value of new life may be zero. If the value of new life is generally zero, it is meaningless to introduce t(0) and c(i), i=1, 2, ⋯, m. However, it depends on the simplicity of our model that the situation where the value of new life is positive can merely appear on the relation between NLSP and QELS. The positive value of new life is a more ordinary situation than the valueless of that. In order to know this fact, we must introdude the concept, that is, the mode of life, which is corresponding to the technology in the production of goods.

5 Mode of Life and Value of Life

We shall remove a constraint of our discussion again. So far we presume that the life process of each generation is expressed by one composition, that is, (D(i), l(i), e(i), c(i)) for i-th generation. We call this composition the mode of life. The word "mode of life" is usually used with various meanings. However, in the investigation of life process, this concept has basically four dimensions. Now, we can suppose that each generation has multiple different modes of life. For example, when there are some groups in which the combination of necessary living goods is different, D(i) is also different among these groups. The difference of l(i) is not propensity for working but the difference of labor time for which capitalists compel laborer to work. We treat it one of difference of mode. The difference of e(i) implies various meanings. As previously mentinned, it may be caused by intensification of working. In general, it is possibly thought that e(i)'s are inversely related to l(i)'s. e(i)'s are also influenced by circumstances of life or customs of life. The difference of c(i)'s is caused complexity. Though it is serious problem for us to investigate causes of the difference of e(i)'s, we put it into the other books.

The mode of life can, superficially, be chosen by voluntary wills of each individual or group. However, it is because the economic principle which regulates the life process is not considered. Even if some generations can choose multiple mode of life, the economic principle discards some of them, which are branded as meaningless modes. Then the purpose of this section is to investigate the relation between the choice problem of mode of life and the value of new life. Then, as mentioned last section, we shall show that the situation where the positive value of new life is not special case but is realistic.

Let (D(i, j), l(i, j), e(i+1, j), c(i, j)) be the mode of life for j-th group of i-th generation. Here, $l(i, j)\geqq 0$, $\neq 0$, $1\geqq e(i, j)>0$, $c(i, j)\geqq 0$ for each i, j, and i-th generation (i=0, 1, 2, ⋯, m) has h^i mode of life. Now the necessary procreation condition is generally specified as follows. Thus, if there exists at least one nontrivial combination of nonnegative y(i, j)'s (i=0, 1, 2, ⋯, m; j=1, 2, ⋯, h^i) which satisfies

$$\sum_{i=1}^{m}\sum_{i=1}^{h^i} c(i, j)y(i, j)\geqq\sum_{i=1}^{h^o} y(0, j) \tag{3}$$

$$\sum_{j=1}^{h^{i-1}} e(i, j)y(i-1, j)\geqq\sum_{i=1}^{h^i} y(i, j) \quad (i=1, 2, \cdots, m), \tag{4}$$

then the necessary procreation condition is fulfilled. Our basic assumption says that this condition is satisfied for every system which we deal with.

Next let us construct problems as a criterion which control this generalized life process. The main problem with procreation is to minimize $\sum D(i, j)y(i, j)$ subject to (3), (4) and

$$\sum_{i=1}^{m}\sum_{j=1}^{h^i} l(i, j)y(i, j)\geqq 1 \tag{5}$$

$$y(i, j)\geqq 0 \quad (i=0, 1, \cdots, m; j=1, 2, \cdots, h^i).$$

Let us call this problem (Mg2) shortly. Furthermore, the problem which is to minimize $\sum D(i, j)y(i, j)$ subject to (4) and (5) is called (Mgl) The latter problem do not involve the procreation. The dual problems of them are called (mg2) and (mgl) respectively.

Effective mode of life (EML) is defined as follows.

Definition 3

In (Mgl), if there is no other solution than what makes y(i, j) strictly positive, then i, j mode of life is called Effective Mode of Life (EML). On the other hand, if i, j mode of life has no positive solution, then it is called Ineffective Mode of Life (IEML).

Moreover, let us generalize ELS as follows.

Definition 4

If among y(i, j)'s ($j=1, 2, \cdots, h^i$) of i-th generation, at leat one element is strictly positive for every solution of (Mg2), then i-th generation iscalled effective generation of labor power (EGLP) and the life span till the oldest generation of them is called effective life span (ELS).

First, let us prove the following lemma.

Lemma 6

Groups that belong to EGLP for (Mgl) and employ IEML can not preserve value on the life process of the dual problem. In other word, when u-th generation belongs to ELS and y(u, v)>0 does not hold for every solution,

$$t(u)+D(u, v)>vl(u, v)+t(u+1)e(u, v) \qquad (6)$$

holds for every dual solution.

proof

We cannot derive this proposition from the dual theorem directly. The theorem merely says that if there exists the solution which satisfies (6), y(u, v) =0 for every solution of the main problem. Thus this proposition has to be proved.

Let us consider some solution of (Mgl). By the presumption of lemma, y(u, v)=0 of the solution. The constraint equation for this generation is,

$$\sum_{j=1}^{h^{u-1}} e(u, j)y(u-1, j) \geqq \sum_{j=1}^{h^u} y(u, j).$$

Since the generation is in ELS, at least one element of y(u−1, j) ($j=1, \cdots, h^{u-1}$) is positive. Then maintaining the inequality sign > or = strictly, we can make y(u, v) positive and increase the positive element of left hand side. Next let us repeat it on,

$$\sum_{j=1}^{h^{u-2}} e(u-1, j)y(u-2, j) \geqq \sum_{j=1}^{h^{u-1}} y(u-1, j).$$

Since one element of the right hand side increases, we can choose one positive element which is increased to maintain this inequality sign or equality sign. Similarly, we can repeat the procedure till the zero generation. Then we denote the new configuration $\hat{y}(i, j)$ ($j=0, 1, \cdots, m$; $j=1, 2, \cdots, h^i$). Except for $\hat{y}(u, v)$, the situation of gign for each element of this configuration is equivalent to the optimal solution. On the other hand, the situation of inequality sign for each constraint equation is completely equivalent to the original solution. Then let us normalize this configuration as follows.

$$\sum_{i=1}^{m}\sum_{j-1}^{h^i} l(i, j)\hat{y}(i, j)=1 \qquad (7)$$

It is clear that we do not lose characteristics mentioned above by this manipulation. Thus this composition of $\hat{y}(i, j)$'s is one of feasible solutions.

The optimal solution of (mgl) satisfies next constraint equations.

$$D(0, j)\geqq t(1)e(1, j) \qquad j=1, 2, \cdots, h^0$$
$$t(i)+D(i, j)\geqq vl(i, j)+t(i+1)e(i+1, j) \qquad i=1, \cdots, m-1; 1, \cdots, h^i$$
$$t(m)+D(m, j)\geqq vl(m, j) \qquad j=1, \cdots, h^m$$

Let us assume that the equation (6) hold equally. Then let multiply the corresponding $\hat{y}(i, j)$'s to above equations respectively and add up for each side. Since $\hat{y}(i, j)$'s are the same arrangement of sign as the optimal solution except for $\hat{y}(u, v)$, when some constraint equation of (mgl) holds with strict inequality, $\hat{y}(i, j)$ for that mode of life is zero. Moreover, for $\hat{y}(u, v)$, the corresponding constraint equation holds with equality. Therefore the sum of each side is also equivalent. Thus,

$$\sum_{i=1}^{m}\sum_{i=1}^{h^i} t(i)\hat{y}(i, j)+\sum_{i=0}^{m}\sum_{j=1}^{h^i} D(i, j)\hat{y}(i, j)$$
$$=v\sum_{i=1}^{m}\sum_{j=1}^{h^i} l(i, j)\hat{y}(i, j)+\sum_{i=1}^{m}\sum_{j=1}^{h^{i-1}} t(i)e(i, j)\hat{y}(i-1, j). \qquad (8)$$

Furthermore, since the constraint equation for each $\hat{y}(i, j)$ preserves the situation of sign of the original solution, when $\sum \hat{y}(i, j)<\sum e(i, j)\hat{y}(i-1, j)$ for some i, $t(i)=0$ holds. Then we have,

$$\sum_{i=1}^{m} t(i)(\sum_{j=1}^{h^i} \hat{y}(i, j)-\sum_{j=1}^{h^{i-1}} e(i, j)\hat{y}(i-1, j))=0.$$

By this and (7), we can finally reform (8) as follows.

$$\sum_{i=0}^{m}\sum_{j=1}^{h^i} D(i, j)\hat{y}(i, j)\ \hat{y}(i, j)=v.$$

Since v is the optimal value of (mgl) and the same as the minimum value of (Mgl), the composition of $\hat{y}(i, j)$'s is the optimal solution of (Mgl). It contradicts our assumption that there is no optimal solution with $y(u, v)>0$. Therefore, (6) has to hold with strict inequality. (Q.E.D)

It may be thought as if this type of proposition can be applicable to another linear programming problem generally. However, it decisively depends on characteristics of this problem that we can construct the configuration of $\hat{y}(i, j)$'s.

By using this lemma, we can easily prove the following theorem.

Theorem 5

If some IEML (ineffective mode of life) of a EGLP (effective generation of labor power) is indispensable to satisfy necessary procreation condition condition, the value of new life is positive

Proof

Let the solution of (mg 2) be $t(i)\ (i=1, \cdots, m)$, v and that of (mg 1) be $t'(i)\ (i=1, \cdots, m)$, v'. Since the solution $t'(i)\ (i=1, \cdots, m)$, v' and $t'(0)=0$ is one of feasible solutions of (mg 2),

$$v \geqq v' \tag{9}$$

has to be satisfied. Let (u, v) mode of life satisfy the presumption of theorem. Since the mode of life is indispensable for procreation, $y(u, v)>0$ for the solution of (Mg 2). Thus, from the duality theorem, we have,

$$t(u)+D(u, v)=vl(u, v)+t(u+1)e(u+1, v)+t(0)c(u, v). \tag{10}$$

On the other hand, from above lemma 6,

$$t'(u)+D(u, v)>v'l((u, v)+t'(u+1)e(u+1,v) \tag{11}$$

is satisfied for every solution of (mg 1). We must pay attention to the fact that if there exists a solution which satisfies this equation with sign of equality, the value of labor power is less than v'.

Let us assume $t(0)=0$ inversely. Then it is clear that the composition of v, $t(i)\ (i=1, \cdots, m)$ is one of feasible solution of (mg 1). Furthermore, since

$$t(u)+D(u, v)=vl(u, v)+t(u+1)e(u+1, v)$$

is satisfied, we have

$$v<v'.$$

It contradicts (9). Therefore $t0)>0$ has to be satisfied. (Q.E.D)

Now we repeat the discussion posed at the last part of the last section. In that discussion, we defined the case in which the value of new life is positive by analyzing the relation between necessary life span of procreation and necessary life span of labor power. However, we felt that the situation where the former is longer than the latter is somewhat unrealistic. On the other hand, the Theorem 5 is realistic in present days. In case that, although some modes of life has strong tendency to supply labor, if the modes of life are employed, it is difficult to have the power of procreation totally, the value of new life will be positive.

6 Concluding Remarks

Throughout the above analysis, we have been concentrate on the system of life process. From the view point of total economic system, it is merely a sub-system. So far, however, this sub-system had been treated as a extremely simplified system. Therefore, we posed a exact system of life process and showed basic characteristics of the system. Even so, we are remaining important studies, that is to say, studies on the relation between the production process of goods and the life process. The studies shall be done in another paper.

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